

Propositional Calculus¹

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Summary. We develop the classical propositional calculus, following [3].

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The articles [4], [1], and [2] provide the notation and terminology for this paper.

We follow the rules: p, q, r, s are elements of CQC-WFF and X is a subset of CQC-WFF.

One can prove the following propositions:

- (1) $(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r)) \in \text{Taut}$.
- (2) If $p \Rightarrow q \in \text{Taut}$, then $(q \Rightarrow r) \Rightarrow (p \Rightarrow r) \in \text{Taut}$.
- (3) If $p \Rightarrow q \in \text{Taut}$ and $q \Rightarrow r \in \text{Taut}$, then $p \Rightarrow r \in \text{Taut}$.
- (4) $p \Rightarrow p \in \text{Taut}$.
- (5) $q \Rightarrow (p \Rightarrow q) \in \text{Taut}$.
- (6) $((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r) \in \text{Taut}$.
- (7) $q \Rightarrow ((q \Rightarrow p) \Rightarrow p) \in \text{Taut}$.
- (8) $(s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p)) \in \text{Taut}$.
- (9) $(q \Rightarrow r) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in \text{Taut}$.
- (10) $(q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r) \in \text{Taut}$.
- (11) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in \text{Taut}$.
- (12) $\neg \text{VERUM} \Rightarrow p \in \text{Taut}$.
- (13) If $q \in \text{Taut}$, then $p \Rightarrow q \in \text{Taut}$.
- (14) If $p \in \text{Taut}$, then $(p \Rightarrow q) \Rightarrow q \in \text{Taut}$.
- (15) If $s \Rightarrow (q \Rightarrow p) \in \text{Taut}$, then $q \Rightarrow (s \Rightarrow p) \in \text{Taut}$.
- (16) If $s \Rightarrow (q \Rightarrow p) \in \text{Taut}$ and $q \in \text{Taut}$, then $s \Rightarrow p \in \text{Taut}$.
- (17) If $s \Rightarrow (q \Rightarrow p) \in \text{Taut}$ and $q \in \text{Taut}$ and $s \in \text{Taut}$, then $p \in \text{Taut}$.
- (18) If $q \Rightarrow (q \Rightarrow r) \in \text{Taut}$, then $q \Rightarrow r \in \text{Taut}$.

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- (19) If $p \Rightarrow (q \Rightarrow r) \in \text{Taut}$, then $(p \Rightarrow q) \Rightarrow (p \Rightarrow r) \in \text{Taut}$.
- (20) If $p \Rightarrow (q \Rightarrow r) \in \text{Taut}$ and $p \Rightarrow q \in \text{Taut}$, then $p \Rightarrow r \in \text{Taut}$.
- (21) If $p \Rightarrow (q \Rightarrow r) \in \text{Taut}$ and $p \Rightarrow q \in \text{Taut}$ and $p \in \text{Taut}$, then $r \in \text{Taut}$.
- (22) If $p \Rightarrow (q \Rightarrow r) \in \text{Taut}$ and $p \Rightarrow (r \Rightarrow s) \in \text{Taut}$, then $p \Rightarrow (q \Rightarrow s) \in \text{Taut}$.
- (23) $p \Rightarrow \text{VERUM} \in \text{Taut}$.
- (24) $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p) \in \text{Taut}$.
- (25) $\neg\neg p \Rightarrow p \in \text{Taut}$.
- (26) $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p) \in \text{Taut}$.
- (27) $p \Rightarrow \neg\neg p \in \text{Taut}$.
- (28) $(\neg\neg p \Rightarrow q) \Rightarrow (p \Rightarrow q) \in \text{Taut}$ and $(p \Rightarrow q) \Rightarrow (\neg\neg p \Rightarrow q) \in \text{Taut}$.
- (29) $(p \Rightarrow \neg\neg q) \Rightarrow (p \Rightarrow q) \in \text{Taut}$ and $(p \Rightarrow q) \Rightarrow (p \Rightarrow \neg\neg q) \in \text{Taut}$.
- (30) $(p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p) \in \text{Taut}$.
- (31) $(\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p) \in \text{Taut}$.
- (32) $(p \Rightarrow \neg p) \Rightarrow \neg p \in \text{Taut}$.
- (33) $\neg p \Rightarrow (p \Rightarrow q) \in \text{Taut}$.
- (34) $p \Rightarrow q \in \text{Taut}$ iff $\neg q \Rightarrow \neg p \in \text{Taut}$.
- (35) If $\neg p \Rightarrow \neg q \in \text{Taut}$, then $q \Rightarrow p \in \text{Taut}$.
- (36) $p \in \text{Taut}$ iff $\neg\neg p \in \text{Taut}$.
- (37) $p \Rightarrow q \in \text{Taut}$ iff $p \Rightarrow \neg\neg q \in \text{Taut}$.
- (38) $p \Rightarrow q \in \text{Taut}$ iff $\neg\neg p \Rightarrow q \in \text{Taut}$.
- (39) If $p \Rightarrow \neg q \in \text{Taut}$, then $q \Rightarrow \neg p \in \text{Taut}$.
- (40) If $\neg p \Rightarrow q \in \text{Taut}$, then $\neg q \Rightarrow p \in \text{Taut}$.
- (41) $\vdash (p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r))$.
- (42) If $\vdash p \Rightarrow q$, then $\vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.
- (43) If $\vdash p \Rightarrow q$ and $\vdash q \Rightarrow r$, then $\vdash p \Rightarrow r$.
- (44) $\vdash p \Rightarrow p$.
- (45) $\vdash p \Rightarrow (q \Rightarrow p)$.
- (46) If $\vdash p$, then $\vdash q \Rightarrow p$.
- (47) $\vdash (s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p))$.
- (48) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow (p \Rightarrow r)$.
- (49) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash q$, then $\vdash p \Rightarrow r$.
- (50) $\vdash p \Rightarrow \text{VERUM}$ and $\vdash \neg\text{VERUM} \Rightarrow p$.
- (51) $\vdash p \Rightarrow ((p \Rightarrow q) \Rightarrow q)$.
- (52) $\vdash (q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r)$.

- (53) If $\vdash q \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow r$.
- (54) $\vdash (p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$.
- (55) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.
- (56) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash p \Rightarrow q$, then $\vdash p \Rightarrow r$.
- (57) $\vdash ((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r)$.
- (58) If $\vdash (p \Rightarrow q) \Rightarrow r$, then $\vdash q \Rightarrow r$.
- (59) $\vdash (p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q))$.
- (60) If $\vdash p \Rightarrow q$, then $\vdash (r \Rightarrow p) \Rightarrow (r \Rightarrow q)$.
- (61) $\vdash (p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$.
- (62) $\vdash (\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$.
- (63) $\vdash \neg p \Rightarrow \neg q$ iff $\vdash q \Rightarrow p$.
- (64) $\vdash p \Rightarrow \neg \neg p$.
- (65) $\vdash \neg \neg p \Rightarrow p$.
- (66) $\vdash \neg \neg p$ iff $\vdash p$.
- (67) $\vdash (\neg \neg p \Rightarrow q) \Rightarrow (p \Rightarrow q)$.
- (68) $\vdash \neg \neg p \Rightarrow q$ iff $\vdash p \Rightarrow q$.
- (69) $\vdash (p \Rightarrow \neg \neg q) \Rightarrow (p \Rightarrow q)$.
- (70) $\vdash p \Rightarrow \neg \neg q$ iff $\vdash p \Rightarrow q$.
- (71) $\vdash (p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p)$.
- (72) If $\vdash p \Rightarrow \neg q$, then $\vdash q \Rightarrow \neg p$.
- (73) $\vdash (\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p)$.
- (74) If $\vdash \neg p \Rightarrow q$, then $\vdash \neg q \Rightarrow p$.
- (75) If $X \vdash p \Rightarrow q$, then $X \vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.
- (76) If $X \vdash p \Rightarrow q$ and $X \vdash q \Rightarrow r$, then $X \vdash p \Rightarrow r$.
- (77) $X \vdash p \Rightarrow p$.
- (78) If $X \vdash p$, then $X \vdash q \Rightarrow p$.
- (79) If $X \vdash p$, then $X \vdash (p \Rightarrow q) \Rightarrow q$.
- (80) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash q \Rightarrow (p \Rightarrow r)$.
- (81) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash q$, then $X \vdash p \Rightarrow r$.
- (82) If $X \vdash p \Rightarrow (p \Rightarrow q)$, then $X \vdash p \Rightarrow q$.
- (83) If $X \vdash (p \Rightarrow q) \Rightarrow r$, then $X \vdash q \Rightarrow r$.
- (84) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.
- (85) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash p \Rightarrow q$, then $X \vdash p \Rightarrow r$.
- (86) $X \vdash \neg p \Rightarrow \neg q$ iff $X \vdash q \Rightarrow p$.

- (87) $X \vdash \neg\neg p$ iff $X \vdash p$.
- (88) $X \vdash p \Rightarrow \neg\neg q$ iff $X \vdash p \Rightarrow q$.
- (89) $X \vdash \neg\neg p \Rightarrow q$ iff $X \vdash p \Rightarrow q$.
- (90) If $X \vdash p \Rightarrow \neg q$, then $X \vdash q \Rightarrow \neg p$.
- (91) If $X \vdash \neg p \Rightarrow q$, then $X \vdash \neg q \Rightarrow p$.
- (92) If $X \vdash p \Rightarrow \neg q$ and $X \vdash q$, then $X \vdash \neg p$.
- (93) If $X \vdash \neg p \Rightarrow q$ and $X \vdash \neg q$, then $X \vdash p$.

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