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Linear Independence in Left Module over Domain[1]

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Summary. Notion of a submodule generated by a set of vectors and linear independence of a set of vectors. A few theorems originated as a generalization of the theorems from the article [10].

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The articles [7], [14], [4], [15], [2], [3], [8], [1], [9], [5], [6], [13], [12], and [11] provide the notation and terminology for this paper.

For simplicity, we use the following convention: x is a set, R is a ring, V is a left module over R, v, v_1, v_2 are vectors of V, and A, B are subsets of V.

Let R be a non empty double loop structure, let V be a non empty vector space structure over R, and let I_1 be a subset of V. We say that I_1 is linearly independent if and only if:

(Def. 1) For every linear combination l of I_1 such that $\sum l = 0_V$ holds the support of $l = \emptyset$.

We introduce I_1 is linearly dependent as an antonym of I_1 is linearly independent.

Next we state several propositions:

(2)[1] If $A \subseteq B$ and B is linearly independent, then A is linearly independent.

(3) If $0_R \neq \mathbf{1}_R$ and A is linearly independent, then $0_V \notin A$.

(4) $\emptyset_{\text{the carrier of } V}$ is linearly independent.

(5) If $0_R \neq \mathbf{1}_R$ and $\{v_1, v_2\}$ is linearly independent, then $v_1 \neq 0_V$ and $v_2 \neq 0_V$.

(6) If $0_R \neq \mathbf{1}_R$, then $\{v, 0_V\}$ is linearly dependent and $\{0_V, v\}$ is linearly dependent.

(7) Let R be an integral domain, V be a left module over R, L be a linear combination of V, and a be a scalar of R. If $a \neq 0_R$, then the support of $a \cdot L =$ the support of L.

(8) Let R be an integral domain, V be a left module over R, L be a linear combination of V, and a be a scalar of R. Then $\sum(a \cdot L) = a \cdot \sum L$.

For simplicity, we adopt the following convention: R is an integral domain, V is a left module over R, A, B are subsets of V, and l is a linear combination of A.

Let us consider R, let us consider V, and let us consider A. The functor $\mathrm{Lin}(A)$ yielding a strict subspace of V is defined by:

(Def. 2) The carrier of $\mathrm{Lin}(A) = \{\sum l\}$.

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[1] The proposition (1) has been removed.

We now state a number of propositions:

(9) $x \in \mathrm{Lin}(A)$ iff there exists l such that $x = \sum l$.

(10) If $x \in A$, then $x \in \mathrm{Lin}(A)$.

(11) $\mathrm{Lin}(\emptyset_{\text{the carrier of } V}) = \mathbf{0}_V$.

(12) If $\mathrm{Lin}(A) = \mathbf{0}_V$, then $A = \emptyset$ or $A = \{0_V\}$.

(13) For every strict subspace W of V such that $0_R \neq \mathbf{1}_R$ and $A =$ the carrier of W holds $\mathrm{Lin}(A) = W$.

(14) Let V be a strict left module over R and A be a subset of V. If $0_R \neq \mathbf{1}_R$ and $A =$ the carrier of V, then $\mathrm{Lin}(A) = V$.

(15) If $A \subseteq B$, then $\mathrm{Lin}(A)$ is a subspace of $\mathrm{Lin}(B)$.

(16) For every strict left module V over R and for all subsets A, B of V such that $\mathrm{Lin}(A) = V$ and $A \subseteq B$ holds $\mathrm{Lin}(B) = V$.

(17) $\mathrm{Lin}(A \cup B) = \mathrm{Lin}(A) + \mathrm{Lin}(B)$.

(18) $\mathrm{Lin}(A \cap B)$ is a subspace of $\mathrm{Lin}(A) \cap \mathrm{Lin}(B)$.

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