JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Gauges

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MML Identifier: JORDAN8.

WWW: http://mizar.org/JFM/Vol11/jordan8.html

The articles [19], [5], [10], [1], [16], [18], [21], [4], [2], [3], [20], [12], [11], [17], [7], [8], [9], [13], [14], [6], and [15] provide the notation and terminology for this paper.

For simplicity, we use the following convention: i, i_1, i_2, j, j_1, j_2, k, m, n are natural numbers, D is a set, f is a finite sequence of elements of D, and G is a matrix over D.

One can prove the following propositions:

(1) If $\mathrm{len}\, f \geq 2$, then $f \restriction 2 = \langle f_1, f_2 \rangle$.

(2) If $k+1 \leq \mathrm{len}\, f$, then $f \restriction (k+1) = (f \restriction k) \frown \langle f_{k+1} \rangle$.

(3) ε_D is a sequence which elements belong to G.

(5)[1] Let D be a non empty set, f be a finite sequence of elements of D, and G be a matrix over D. Suppose f is a sequence which elements belong to G. Then $f_{\lfloor m}$ is a sequence which elements belong to G.

(6) Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then there exist natural numbers i_1, j_1, i_2, j_2 such that

(i) $\langle i_1, j_1 \rangle \in$ the indices of G,

(ii) $f_k = G \circ (i_1, j_1)$,

(iii) $\langle i_2, j_2 \rangle \in$ the indices of G,

(iv) $f_{k+1} = G \circ (i_2, j_2)$, and

(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ or $i_1 + 1 = i_2$ and $j_1 = j_2$ or $i_1 = i_2 + 1$ and $j_1 = j_2$ or $i_1 = i_2$ and $j_1 = j_2 + 1$.

In the sequel G is a Go-board and p is a point of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state several propositions:

(7) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a sequence which elements belong to G. Then f is standard and special.

(8) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $\mathrm{len}\, f \geq 2$ and f is a sequence which elements belong to G. Then f is non constant.

[1] The proposition (4) has been removed.

(9) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) f is a sequence which elements belong to G,

(ii) there exist i, j such that $\langle i, j\rangle \in$ the indices of G and $p = G\circ(i, j)$, and

(iii) for all i_1, j_1, i_2, j_2 such that $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_{{\rm len}\, f} = G\circ(i_1, j_1)$ and $p = G\circ(i_2, j_2)$ holds $|i_2 - i_1| + |j_2 - j_1| = 1$.

Then $f \frown \langle p\rangle$ is a sequence which elements belong to G.

(10) If $i+k < {\rm len}\, G$ and $1 \leq j$ and $j < {\rm width}\, G$ and ${\rm cell}(G, i, j)$ meets ${\rm cell}(G, i+k, j)$, then $k \leq 1$.

(11) For every non empty compact subset C of $\mathcal{E}^2_{\rm T}$ holds C is vertical iff ${\rm E\text{-}bound}(C) \leq {\rm W\text{-}bound}(C)$.

(12) For every non empty compact subset C of $\mathcal{E}^2_{\rm T}$ holds C is horizontal iff ${\rm N\text{-}bound}(C) \leq {\rm S\text{-}bound}(C)$.

Let C be a subset of $\mathcal{E}^2_{\rm T}$ and let n be a natural number. The functor ${\rm Gauge}(C,n)$ yields a matrix over $\mathcal{E}^2_{\rm T}$ and is defined by the conditions (Def. 1).

(Def. 1)(i) ${\rm len}\, {\rm Gauge}(C,n) = 2^n + 3$,

(ii) ${\rm len}\, {\rm Gauge}(C,n) = {\rm width}\, {\rm Gauge}(C,n)$, and

(iii) for all i, j such that $\langle i, j\rangle \in$ the indices of ${\rm Gauge}(C,n)$ holds ${\rm Gauge}(C,n)\circ(i, j) = [{\rm W\text{-}bound}(C) + \frac{{\rm E\text{-}bound}(C) - {\rm W\text{-}bound}(C)}{2^n}\cdot(i-2), {\rm S\text{-}bound}(C) + \frac{{\rm N\text{-}bound}(C) - {\rm S\text{-}bound}(C)}{2^n}\cdot(j-2)]$.

Let C be a non empty subset of $\mathcal{E}^2_{\rm T}$ and let n be a natural number. Note that ${\rm Gauge}(C,n)$ is non empty yielding, line **X**-constant, and column **Y**-constant.

Let C be a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\rm T}$ and let us consider n. Note that ${\rm Gauge}(C,n)$ is line **Y**-increasing and column **X**-increasing.

In the sequel T is a non empty subset of $\mathcal{E}^2_{\rm T}$.

Next we state several propositions:

(13) ${\rm len}\, {\rm Gauge}(T,n) \geq 4$.

(14) If $1 \leq j$ and $j \leq {\rm len}\, {\rm Gauge}(T,n)$, then $({\rm Gauge}(T,n)\circ(2, j))_{\mathbf{1}} = {\rm W\text{-}bound}(T)$.

(15) If $1 \leq j$ and $j \leq {\rm len}\, {\rm Gauge}(T,n)$, then $({\rm Gauge}(T,n)\circ({\rm len}\, {\rm Gauge}(T,n) -' 1, j))_{\mathbf{1}} = {\rm E\text{-}bound}(T)$.

(16) If $1 \leq i$ and $i \leq {\rm len}\, {\rm Gauge}(T,n)$, then $({\rm Gauge}(T,n)\circ(i,2))_{\mathbf{2}} = {\rm S\text{-}bound}(T)$.

(17) If $1 \leq i$ and $i \leq {\rm len}\, {\rm Gauge}(T,n)$, then $({\rm Gauge}(T,n)\circ(i, {\rm len}\, {\rm Gauge}(T,n) -' 1))_{\mathbf{2}} = {\rm N\text{-}bound}(T)$.

In the sequel C denotes a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\rm T}$.

The following four propositions are true:

(18) If $i \leq {\rm len}\, {\rm Gauge}(C,n)$, then ${\rm cell}({\rm Gauge}(C,n), i, {\rm len}\, {\rm Gauge}(C,n))$ misses C.

(19) If $j \leq {\rm len}\, {\rm Gauge}(C,n)$, then ${\rm cell}({\rm Gauge}(C,n), {\rm len}\, {\rm Gauge}(C,n), j)$ misses C.

(20) If $i \leq {\rm len}\, {\rm Gauge}(C,n)$, then ${\rm cell}({\rm Gauge}(C,n), i, 0)$ misses C.

(21) If $j \leq {\rm len}\, {\rm Gauge}(C,n)$, then ${\rm cell}({\rm Gauge}(C,n), 0, j)$ misses C.

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Received January 22, 1999

Published January 2, 2004