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Projections in n-Dimensional Euclidean Space to Each Coordinates

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Summary. In the n-dimensional Euclidean space $\mathcal{E}^n_{\rm T}$, a projection operator to each coordinate is defined. It is proven that such an operator is linear. Moreover, it is continuous as a mapping from $\mathcal{E}^n_{\rm T}$ to R^1, the carrier of which is a set of all reals. If n is 1, the projection becomes a homeomorphism, which means that $\mathcal{E}^1_{\rm T}$ is homeomorphic to R^1.

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The articles [22], [27], [2], [24], [16], [1], [26], [10], [21], [28], [3], [12], [7], [8], [6], [25], [4], [15], [14], [20], [23], [17], [13], [18], [9], [19], [11], and [5] provide the notation and terminology for this paper.

1. Projections

For simplicity, we use the following convention: s_1, r, r_1, r_2 denote real numbers, s denotes a real number, n, i denote natural numbers, X denotes a non empty topological space, p, p_1, p_2 denote points of $\mathcal{E}^n_{\rm T}$, and P denotes a subset of $\mathcal{E}^n_{\rm T}$.

Let n, i be natural numbers and let p be an element of $\mathcal{E}^n_{\rm T}$. The functor $\mathrm{Proj}(p,i)$ yields a real number and is defined as follows:

(Def. 1) For every finite sequence g of elements of $\mathbb{R}$ such that $g = p$ holds $\mathrm{Proj}(p,i) = g_i$.

The following propositions are true:

(1) There exists a map f from $\mathcal{E}^n_{\rm T}$ into $\mathbb{R}^{\mathbf{1}}$ such that for every element p of $\mathcal{E}^n_{\rm T}$ holds $f(p) = \mathrm{Proj}(p,i)$.

(2) For every i such that $i \in \mathrm{Seg}\, n$ holds $\langle \underbrace{0,\ldots,0}_{n} \rangle(i) = 0$.

(3) For every i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(0_{\mathcal{E}^n_{\rm T}}, i) = 0$.

(4) For all r, p, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(r \cdot p, i) = r \cdot \mathrm{Proj}(p,i)$.

(5) For all p, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(-p,i) = -\mathrm{Proj}(p,i)$.

(6) For all p_1, p_2, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(p_1 + p_2, i) = \mathrm{Proj}(p_1,i) + \mathrm{Proj}(p_2,i)$.

(7) For all p_1, p_2, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(p_1 - p_2, i) = \mathrm{Proj}(p_1,i) - \mathrm{Proj}(p_2,i)$.

(8) $\mathrm{len}\langle\underbrace{0,\ldots,0}_{n}\rangle = n$.

(9) If $i \leq n$, then $\langle\underbrace{0,\ldots,0}_{n}\rangle{\restriction}i = \langle\underbrace{0,\ldots,0}_{i}\rangle$.

(10) $\langle\underbrace{0,\ldots,0}_{n}\rangle_{\lfloor i} = \langle\underbrace{0,\ldots,0}_{n -' i}\rangle$.

(11) $\sum\langle\underbrace{0,\ldots,0}_{i}\rangle = 0$.

(12) For every finite sequence w and for every set r and for every i holds $\mathrm{len}(w +\cdot (i,r)) = \mathrm{len}\, w$.

(13) Let D be a non empty set, w be a finite sequence of elements of D, and r be an element of D. If $i \in \mathrm{dom}\, w$, then $w +\cdot (i,r) = (w{\restriction}(i -' 1)) \frown \langle r\rangle \frown (w_{\lfloor i})$.

(14) For every real number r such that $i \in \mathrm{Seg}\, n$ holds $\sum(\langle\underbrace{0,\ldots,0}_{n}\rangle +\cdot (i,r)) = r$.

(15) For every element q of $\mathcal{R}^n$ and for all p, i such that $i \in \mathrm{Seg}\, n$ and $q = p$ holds $\mathrm{Proj}(p,i) \leq |q|$ and $(\mathrm{Proj}(p,i))^{\mathbf{2}} \leq |q|^{\mathbf{2}}$.

2. Continuity of Projections

One can prove the following propositions:

(16) For all s_1, P, i such that $P = \{p : s_1 > \mathrm{Proj}(p,i)\}$ and $i \in \mathrm{Seg}\, n$ holds P is open.

(17) For all s_1, P, i such that $P = \{p : s_1 < \mathrm{Proj}(p,i)\}$ and $i \in \mathrm{Seg}\, n$ holds P is open.

(18) Let P be a subset of $\mathcal{E}^n_{\mathrm{T}}$, a, b be real numbers, and given i. Suppose $P = \{p; p$ ranges over elements of $\mathcal{E}^n_{\mathrm{T}}$: $a < \mathrm{Proj}(p,i) \wedge \mathrm{Proj}(p,i) < b\}$ and $i \in \mathrm{Seg}\, n$. Then P is open.

(19) Let a, b be real numbers, f be a map from $\mathcal{E}^n_{\mathrm{T}}$ into $\mathbb{R}^{\mathbf{1}}$, and given i. Suppose that for every element p of $\mathcal{E}^n_{\mathrm{T}}$ holds $f(p) = \mathrm{Proj}(p,i)$. Then $f^{-1}(\{s : a < s \wedge s < b\}) = \{p; p$ ranges over elements of $\mathcal{E}^n_{\mathrm{T}}$: $a < \mathrm{Proj}(p,i) \wedge \mathrm{Proj}(p,i) < b\}$.

(20) Let M be a non empty metric space and f be a map from X into M_{top}. Suppose that for every real number r and for every element u of the carrier of M and for every subset P of M_{top} such that $r > 0$ and $P = \mathrm{Ball}(u,r)$ holds $f^{-1}(P)$ is open. Then f is continuous.

(21) Let u be a point of the metric space of real numbers and r, u_1 be real numbers. If $u_1 = u$ and $r > 0$, then $\mathrm{Ball}(u,r) = \{s : u_1 - r < s \wedge s < u_1 + r\}$.

(22) Let f be a map from $\mathcal{E}^n_{\mathrm{T}}$ into $\mathbb{R}^{\mathbf{1}}$ and given i. Suppose $i \in \mathrm{Seg}\, n$ and for every element p of $\mathcal{E}^n_{\mathrm{T}}$ holds $f(p) = \mathrm{Proj}(p,i)$. Then f is continuous.

3. 1-dimensional and 2-dimensional Cases

One can prove the following three propositions:

(23) For every s holds $|\langle s\rangle| = \langle |s| \rangle$.

(24) For every element p of $\mathcal{E}^1_{\mathrm{T}}$ there exists a real number r such that $p = \langle r\rangle$.

(25) For every element w of $\mathcal{E}^1$ there exists a real number r such that $w = \langle r\rangle$.

Let r be a real number. The functor $|[r]|$ yielding a point of $\mathcal{E}^1_{\mathrm{T}}$ is defined by:

(Def. 2) $|[r]| = \langle r\rangle$.

We now state a number of propositions:

(26) $s \cdot |[r]| = |[s \cdot r]|$.

(27) $|[r_1 + r_2]| = |[r_1]| + |[r_2]|$.

(28) $|[0]| = 0_{\mathcal{E}^1_{\mathrm{T}}}$.

(29) If $|[r_1]| = |[r_2]|$, then $r_1 = r_2$.

(30) For every subset P of $\mathbb{R}^{\mathbf{1}}$ and for every real number b such that $P = \{s : s < b\}$ holds P is open.

(31) For every subset P of $\mathbb{R}^{\mathbf{1}}$ and for every real number a such that $P = \{s : a < s\}$ holds P is open.

(32) For every subset P of $\mathbb{R}^{\mathbf{1}}$ and for all real numbers a, b such that $P = \{s : a < s \wedge s < b\}$ holds P is open.

(33) For every point u of $\mathcal{E}^1$ and for all real numbers r, u_1 such that $\langle u_1 \rangle = u$ and $r > 0$ holds $\mathrm{Ball}(u, r) = \{\langle s \rangle : u_1 - r < s \wedge s < u_1 + r\}$.

(34) For every map f from $\mathcal{E}^1_{\mathrm{T}}$ into $\mathbb{R}^{\mathbf{1}}$ such that for every element p of $\mathcal{E}^1_{\mathrm{T}}$ holds $f(p) = \mathrm{Proj}(p, 1)$ holds f is a homeomorphism.

(35) For every element p of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{Proj}(p, 1) = p_{\mathbf{1}}$ and $\mathrm{Proj}(p, 2) = p_{\mathbf{2}}$.

(36) For every element p of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{Proj}(p, 1) = \mathrm{proj1}(p)$ and $\mathrm{Proj}(p, 2) = \mathrm{proj2}(p)$.

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