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Some Remarks on Finite Sequences on Go-Boards[1]

Adam Naumowicz
University of Białystok

Summary. This paper shows some properties of finite sequences on Go-boards. It also provides the partial correspondence between two ways of decomposition of curves induced by cages.

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The articles [21], [1], [20], [24], [3], [4], [2], [23], [22], [12], [19], [8], [18], [9], [10], [11], [14], [16], [6], [17], [5], [7], [13], and [15] provide the notation and terminology for this paper.

We adopt the following rules: i, j, k, n are natural numbers, f is a finite sequence of elements of the carrier of $\mathcal{E}^2_{\rm T}$, and G is a Go-board.

One can prove the following propositions:

(1) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (i, j), G \circ (i,k))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle i, j\rangle \in$ the indices of G,

(iv) $\langle i, k\rangle \in$ the indices of G, and

(v) $j \leq k$.

Then there exists n such that $j \leq n$ and $n \leq k$ and $(G \circ (i,n))_{\bf 2} = \inf({\rm proj2}^\circ(\mathcal{L}(G \circ (i, j), G \circ (i,k)) \cap \widetilde{\mathcal{L}}(f)))$.

(2) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (i, j), G \circ (i,k))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle i, j\rangle \in$ the indices of G,

(iv) $\langle i, k\rangle \in$ the indices of G, and

(v) $j \leq k$.

Then there exists n such that $j \leq n$ and $n \leq k$ and $(G \circ (i,n))_{\bf 2} = \sup({\rm proj2}^\circ(\mathcal{L}(G \circ (i, j), G \circ (i,k)) \cap \widetilde{\mathcal{L}}(f)))$.

(3) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (j,i), G \circ (k,i))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle j, i\rangle \in$ the indices of G,

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(iv) $\langle k, i\rangle \in$ the indices of G, and

(v) $j \leq k$.

Then there exists n such that $j \leq n$ and $n \leq k$ and $(G \circ (n,i))_{\mathbf{1}} = \inf(\mathrm{proj1}^{\circ}(\mathcal{L}(G \circ (j,i), G \circ (k,i)) \cap \widetilde{\mathcal{L}}(f)))$.

(4) Suppose that

(i) f is a sequence which elements belong to G,

(ii) $\mathcal{L}(G \circ (j,i), G \circ (k,i))$ meets $\widetilde{\mathcal{L}}(f)$,

(iii) $\langle j, i\rangle \in$ the indices of G,

(iv) $\langle k, i\rangle \in$ the indices of G, and

(v) $j \leq k$.

Then there exists n such that $j \leq n$ and $n \leq k$ and $(G \circ (n,i))_{\mathbf{1}} = \sup(\mathrm{proj1}^{\circ}(\mathcal{L}(G \circ (j,i), G \circ (k,i)) \cap \widetilde{\mathcal{L}}(f)))$.

(5) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{UpperSeq}(C,n))_1 = \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(6) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{LowerSeq}(C,n))_1 = \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(7) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{UpperSeq}(C,n))_{\mathrm{len}\,\mathrm{UpperSeq}(C,n)} = \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(8) For every compact non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ and for every natural number n holds $(\mathrm{LowerSeq}(C,n))_{\mathrm{len}\,\mathrm{LowerSeq}(C,n)} = \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(9) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ or $\widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

We adopt the following convention: C is a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ satisfying conditions of simple closed curve, p is a point of $\mathcal{E}^2_{\mathrm{T}}$, and i_1, j_1, i_2, j_2 are natural numbers.

One can prove the following four propositions:

(10) Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\mathrm{UpperSeq}(C,n)$ is a sequence which elements belong to $\mathrm{Gauge}(C,n)$.

(11) Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) f is a sequence which elements belong to G,

(ii) there exist i, j such that $\langle i, j\rangle \in$ the indices of G and $p = G \circ (i,j)$, and

(iii) for all i_1, j_1, i_2, j_2 such that $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $p = G \circ (i_1, j_1)$ and $f_1 = G \circ (i_2, j_2)$ holds $|i_2 - i_1| + |j_2 - j_1| = 1$.

Then $\langle p\rangle \frown f$ is a sequence which elements belong to G.

(12) Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Then $\mathrm{LowerSeq}(C,n)$ is a sequence which elements belong to $\mathrm{Gauge}(C,n)$.

(13) Suppose $p_{\mathbf{1}} = \frac{\text{W-bound}(C)+\text{E-bound}(C)}{2}$ and $p_{\mathbf{2}} = \inf(\mathrm{proj2}^{\circ}(\mathcal{L}(\mathrm{Gauge}(C,1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,1),1), \mathrm{Gauge}(C,1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,1), \mathrm{width}\,\mathrm{Gauge}(C,1))) \cap \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,i+1)))))$. Then there exists j such that $1 \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,i+1)$ and $p = \mathrm{Gauge}(C,i+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,i+1), j)$.

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