

# Some Properties of Cells and Arcs<sup>1</sup>

Robert Milewski  
University of Białystok

Andrzej Trybulec  
University of Białystok

Artur Korniłowicz  
University of Białystok

Adam Naumowicz  
University of Białystok

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The articles [28], [33], [2], [30], [15], [1], [34], [6], [4], [32], [7], [16], [5], [29], [27], [14], [3], [25], [26], [22], [11], [12], [9], [31], [18], [13], [21], [19], [20], [8], [24], [23], [10], and [17] provide the notation and terminology for this paper.

For simplicity, we use the following convention:  $E$  denotes a compact non vertical non horizontal subset of  $\mathcal{E}_T^2$ ,  $C$  denotes a compact connected non vertical non horizontal subset of  $\mathcal{E}_T^2$ ,  $G$  denotes a Go-board,  $i, j, m, n$  denote natural numbers, and  $p$  denotes a point of  $\mathcal{E}_T^2$ .

Let us note that every simple closed curve is non vertical and non horizontal.

Let  $T$  be a non empty topological space. One can check that there exists a union of components of  $T$  which is non empty.

We now state a number of propositions:

- (1) Let  $T$  be a non empty topological space and  $A$  be a non empty union of components of  $T$ . If  $A$  is connected, then  $A$  is a component of  $T$ .
- (2) For every finite sequence  $f$  holds  $f$  is empty iff  $\text{Rev}(f)$  is empty.
- (3) Let  $D$  be a non empty set,  $f$  be a finite sequence of elements of  $D$ , and given  $i, j$ . If  $1 \leq i$  and  $i \leq \text{len } f$  and  $1 \leq j$  and  $j \leq \text{len } f$ , then  $\text{mid}(f, i, j)$  is non empty.
- (4) Let  $f$  be a non empty finite sequence of elements of  $\mathcal{E}_T^2$  and  $p$  be a point of  $\mathcal{E}_T^2$ . If  $1 \leq \text{len } f$  and  $p \in \tilde{\mathcal{L}}(f)$ , then  $(\downarrow f, p)(1) = f(1)$ .
- (5) Let  $f$  be a non empty finite sequence of elements of  $\mathcal{E}_T^2$  and  $p$  be a point of  $\mathcal{E}_T^2$ . If  $f$  is a special sequence and  $p \in \tilde{\mathcal{L}}(f)$ , then  $(\downarrow p, f)(\text{len } \downarrow p, f) = f(\text{len } f)$ .
- (6) For every simple closed curve  $P$  holds  $W_{\max}(P) \neq E_{\max}(P)$ .
- (7) Let  $D$  be a non empty set and  $f$  be a finite sequence of elements of  $D$ . If  $1 \leq i$  and  $i < \text{len } f$ , then  $(\text{mid}(f, i, \text{len } f - 1)) \cap \langle f_{\text{len } f} \rangle = \text{mid}(f, i, \text{len } f)$ .
- (8) For all points  $p, q$  of  $\mathcal{E}_T^2$  such that  $p \neq q$  and  $\mathcal{L}(p, q)$  is vertical holds  $\langle p, q \rangle$  is a special sequence.
- (9) For all points  $p, q$  of  $\mathcal{E}_T^2$  such that  $p \neq q$  and  $\mathcal{L}(p, q)$  is horizontal holds  $\langle p, q \rangle$  is a special sequence.

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- (10) Let  $p, q$  be finite sequences of elements of  $\mathcal{E}_T^2$  and  $v$  be a point of  $\mathcal{E}_T^2$ . If  $p$  is in the area of  $q$ , then  $p \circ v$  is in the area of  $q$ .
- (11) For every non trivial finite sequence  $p$  of elements of  $\mathcal{E}_T^2$  and for every point  $v$  of  $\mathcal{E}_T^2$  holds  $p \circ v$  is in the area of  $p$ .
- (12) For every finite sequence  $f$  holds  $\text{Center } f \geq 1$ .
- (13) For every finite sequence  $f$  such that  $\text{len } f \geq 1$  holds  $\text{Center } f \leq \text{len } f$ .
- (14)  $\text{Center } G \leq \text{len } G$ .
- (15) For every finite sequence  $f$  such that  $\text{len } f \geq 2$  holds  $\text{Center } f > 1$ .
- (16) For every finite sequence  $f$  such that  $\text{len } f \geq 3$  holds  $\text{Center } f < \text{len } f$ .
- (17)  $\text{Center Gauge}(E, n) = 2^{n-1} + 2$ .
- (18)  $E \subseteq \text{cell}(\text{Gauge}(E, 0), 2, 2)$ .
- (19)  $\text{cell}(\text{Gauge}(E, 0), 2, 2) \not\subseteq \text{BDD}E$ .
- (20)  $\text{Gauge}(C, 1) \circ (\text{Center Gauge}(C, 1), 1) = \left[ \frac{\text{W-bound}(C) + \text{E-bound}(C)}{2}, \text{S-bound}(\tilde{\mathcal{L}}(\text{Cage}(C, 1))) \right]$ .
- (21)  $\text{Gauge}(C, 1) \circ (\text{Center Gauge}(C, 1), \text{len Gauge}(C, 1)) = \left[ \frac{\text{W-bound}(C) + \text{E-bound}(C)}{2}, \text{N-bound}(\tilde{\mathcal{L}}(\text{Cage}(C, 1))) \right]$ .
- (22) If  $1 \leq j$  and  $j < \text{width } G$  and  $1 \leq m$  and  $m \leq \text{len } G$  and  $1 \leq n$  and  $n \leq \text{width } G$  and  $p \in \text{cell}(G, \text{len } G, j)$  and  $p_1 = (G \circ (m, n))_1$ , then  $\text{len } G = m$ .
- (23) Suppose  $1 \leq i$  and  $i \leq \text{len } G$  and  $1 \leq j$  and  $j < \text{width } G$  and  $1 \leq m$  and  $m \leq \text{len } G$  and  $1 \leq n$  and  $n \leq \text{width } G$  and  $p \in \text{cell}(G, i, j)$  and  $p_1 = (G \circ (m, n))_1$ . Then  $i = m$  or  $i = m - 1$ .
- (24) If  $1 \leq i$  and  $i < \text{len } G$  and  $1 \leq m$  and  $m \leq \text{len } G$  and  $1 \leq n$  and  $n \leq \text{width } G$  and  $p \in \text{cell}(G, i, \text{width } G)$  and  $p_2 = (G \circ (m, n))_2$ , then  $\text{width } G = n$ .
- (25) Suppose  $1 \leq i$  and  $i < \text{len } G$  and  $1 \leq j$  and  $j \leq \text{width } G$  and  $1 \leq m$  and  $m \leq \text{len } G$  and  $1 \leq n$  and  $n \leq \text{width } G$  and  $p \in \text{cell}(G, i, j)$  and  $p_2 = (G \circ (m, n))_2$ . Then  $j = n$  or  $j = n - 1$ .
- (26) For every simple closed curve  $C$  and for every real number  $r$  such that  $\text{W-bound}(C) \leq r$  and  $r \leq \text{E-bound}(C)$  holds  $\mathcal{L}([r, \text{S-bound}(C)], [r, \text{N-bound}(C)])$  meets  $\text{UpperArc}(C)$ .
- (27) For every simple closed curve  $C$  and for every real number  $r$  such that  $\text{W-bound}(C) \leq r$  and  $r \leq \text{E-bound}(C)$  holds  $\mathcal{L}([r, \text{S-bound}(C)], [r, \text{N-bound}(C)])$  meets  $\text{LowerArc}(C)$ .
- (28) Let  $C$  be a simple closed curve and  $i$  be a natural number. If  $1 < i$  and  $i < \text{len Gauge}(C, n)$ , then  $\mathcal{L}(\text{Gauge}(C, n) \circ (i, 1), \text{Gauge}(C, n) \circ (i, \text{len Gauge}(C, n)))$  meets  $\text{UpperArc}(C)$ .
- (29) Let  $C$  be a simple closed curve and  $i$  be a natural number. If  $1 < i$  and  $i < \text{len Gauge}(C, n)$ , then  $\mathcal{L}(\text{Gauge}(C, n) \circ (i, 1), \text{Gauge}(C, n) \circ (i, \text{len Gauge}(C, n)))$  meets  $\text{LowerArc}(C)$ .
- (30) For every simple closed curve  $C$  holds  $\mathcal{L}(\text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), 1), \text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), \text{len Gauge}(C, n)))$  meets  $\text{UpperArc}(C)$ .
- (31) For every simple closed curve  $C$  holds  $\mathcal{L}(\text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), 1), \text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), \text{len Gauge}(C, n)))$  meets  $\text{LowerArc}(C)$ .
- (32) Let  $C$  be a compact connected non vertical non horizontal subset of  $\mathcal{E}_T^2$  and  $i$  be a natural number. If  $1 \leq i$  and  $i \leq \text{len Gauge}(C, n)$ , then  $\mathcal{L}(\text{Gauge}(C, n) \circ (i, 1), \text{Gauge}(C, n) \circ (i, \text{len Gauge}(C, n)))$  meets  $\text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ .
- (33) Let  $C$  be a compact connected non vertical non horizontal subset of  $\mathcal{E}_T^2$  and  $i$  be a natural number. If  $1 \leq i$  and  $i \leq \text{len Gauge}(C, n)$ , then  $\mathcal{L}(\text{Gauge}(C, n) \circ (i, 1), \text{Gauge}(C, n) \circ (i, \text{len Gauge}(C, n)))$  meets  $\text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ .

- (34) For every compact connected non vertical non horizontal subset  $C$  of  $\mathcal{E}_T^2$  holds  $\mathcal{L}(\text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), 1), \text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), \text{len Gauge}(C, n)))$  meets  $\text{UpperArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ .
- (35) For every compact connected non vertical non horizontal subset  $C$  of  $\mathcal{E}_T^2$  holds  $\mathcal{L}(\text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), 1), \text{Gauge}(C, n) \circ (\text{Center Gauge}(C, n), \text{len Gauge}(C, n)))$  meets  $\text{LowerArc}(\tilde{\mathcal{L}}(\text{Cage}(C, n)))$ .
- (36) If  $j \leq \text{width } G$ , then  $\text{cell}(G, 0, j)$  is not Bounded.
- (37) If  $i \leq \text{width } G$ , then  $\text{cell}(G, \text{len } G, i)$  is not Bounded.
- (38) If  $j \leq \text{width Gauge}(C, n)$ , then  $\text{cell}(\text{Gauge}(C, n), 0, j) \subseteq \text{UBDC}$ .
- (39) If  $j \leq \text{len Gauge}(E, n)$ , then  $\text{cell}(\text{Gauge}(E, n), \text{len Gauge}(E, n), j) \subseteq \text{UBDE}$ .
- (40) If  $i \leq \text{len Gauge}(C, n)$  and  $j \leq \text{width Gauge}(C, n)$  and  $\text{cell}(\text{Gauge}(C, n), i, j) \subseteq \text{BDDC}$ , then  $j > 1$ .
- (41) If  $i \leq \text{len Gauge}(C, n)$  and  $j \leq \text{width Gauge}(C, n)$  and  $\text{cell}(\text{Gauge}(C, n), i, j) \subseteq \text{BDDC}$ , then  $i > 1$ .
- (42) If  $i \leq \text{len Gauge}(C, n)$  and  $j \leq \text{width Gauge}(C, n)$  and  $\text{cell}(\text{Gauge}(C, n), i, j) \subseteq \text{BDDC}$ , then  $j + 1 < \text{width Gauge}(C, n)$ .
- (43) If  $i \leq \text{len Gauge}(C, n)$  and  $j \leq \text{width Gauge}(C, n)$  and  $\text{cell}(\text{Gauge}(C, n), i, j) \subseteq \text{BDDC}$ , then  $i + 1 < \text{len Gauge}(C, n)$ .
- (44) If there exist  $i, j$  such that  $i \leq \text{len Gauge}(C, n)$  and  $j \leq \text{width Gauge}(C, n)$  and  $\text{cell}(\text{Gauge}(C, n), i, j) \subseteq \text{BDDC}$ , then  $n \geq 1$ .

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