JOURNAL OF FORMALIZED MATHEMATICS
Volume 14, Released 2002, Published 2003
Inst. of Computer Science, Univ. of Białystok

# Properties of the Internal Approximation of Jordan's Curve[1]

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MML Identifier: JORDAN14.

WWW: http://mizar.org/JFM/Vol14/jordan14.html

The articles [19], [25], [14], [10], [1], [16], [2], [3], [24], [11], [18], [9], [26], [6], [17], [7], [8], [12], [13], [20], [15], [4], [5], [21], [23], and [22] provide the notation and terminology for this paper.

The following propositions are true:

(1) For every non constant standard special circular sequence $f$ holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(2) For every non constant standard special circular sequence $f$ holds $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(3) Let $G$ be a Go-board, $f$ be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and $k$ be a natural number. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $f$ is a sequence which elements belong to $G$. Then $\mathrm{left\_cell}(f,k,G)$ is closed.

(4) Let $G$ be a Go-board, $p$ be a point of $\mathcal{E}^2_{\mathrm{T}}$, and $i$, $j$ be natural numbers. Suppose $1 \leq i$ and $i+1 \leq \mathrm{len}\, G$ and $1 \leq j$ and $j+1 \leq \mathrm{width}\, G$. Then $p \in \mathrm{Int}\,\mathrm{cell}(G,i,j)$ if and only if the following conditions are satisfied:

(i) $(G \circ (i,j))_{\mathbf{1}} < p_{\mathbf{1}}$,

(ii) $p_{\mathbf{1}} < (G \circ (i+1,j))_{\mathbf{1}}$,

(iii) $(G \circ (i,j))_{\mathbf{2}} < p_{\mathbf{2}}$, and

(iv) $p_{\mathbf{2}} < (G \circ (i,j+1))_{\mathbf{2}}$.

(5) For every non constant standard special circular sequence $f$ holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

Let $f$ be a non constant standard special circular sequence. One can check that $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

Let $C$ be a simple closed curve and let $n$ be a natural number. The functor $\mathrm{SpanStart}(C,n)$ yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 1) $\mathrm{SpanStart}(C,n) = \mathrm{Gauge}(C,n) \circ (\mathrm{X\text{-}SpanStart}(C,n), \mathrm{Y\text{-}SpanStart}(C,n))$.

We now state four propositions:

(6) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $(\mathrm{Span}(C,n))_1 = \mathrm{SpanStart}(C,n)$.

[1]This work has been partially supported by CALCULEMUS grant HPRN-CT-2000-00102.

(7) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\mathrm{SpanStart}(C,n) \in \mathrm{BDD}\,C$.

(8) Let $C$ be a simple closed curve and $n$, $k$ be natural numbers. Suppose $n$ is sufficiently large for $C$. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\,\mathrm{Span}(C,n)$. Then $\mathrm{right\_cell}(\mathrm{Span}(C,n),k,\mathrm{Gauge}(C,n))$ misses $C$ and $\mathrm{left\_cell}(\mathrm{Span}(C,n),k,\mathrm{Gauge}(C,n))$ meets $C$.

(9) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $C$ misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

Let $C$ be a simple closed curve and let $n$ be a natural number. Observe that $\overline{\mathrm{RightComp}(\mathrm{Span}(C,n))}$ is compact.

One can prove the following propositions:

(10) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $C$ meets $\mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(11) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $C$ misses $\mathrm{RightComp}(\mathrm{Span}(C,n))$.

(12) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(13) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $C \subseteq \mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(14) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(15) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\mathrm{UBD}\,C \subseteq \mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(16) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\mathrm{RightComp}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(17) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\mathrm{UBD}\,C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(18) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $\mathrm{UBD}\,C$ misses $\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(19) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $\mathrm{UBD}\,C$ misses $\mathrm{RightComp}(\mathrm{Span}(C,n))$.

(20) Let $C$ be a simple closed curve, $P$ be a subset of $\mathcal{E}^2_{\mathrm{T}}$, and $n$ be a natural number. Suppose $n$ is sufficiently large for $C$. If $P$ is outside component of $C$, then $P$ misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(21) Let $C$ be a simple closed curve and $n$ be a natural number. If $n$ is sufficiently large for $C$, then $\mathrm{UBD}\,C$ misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(22) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(23) Let $C$ be a simple closed curve and $i$, $j$, $k$, $n$ be natural numbers. Suppose $n$ is sufficiently large for $C$ and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i > 1$.

(24) Let $C$ be a simple closed curve and $i$, $j$, $k$, $n$ be natural numbers. Suppose $n$ is sufficiently large for $C$ and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$.

(25) Let $C$ be a simple closed curve and $i$, $j$, $k$, $n$ be natural numbers. Suppose $n$ is sufficiently large for $C$ and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $j > 1$.

(26) Let $C$ be a simple closed curve and $i$, $j$, $k$, $n$ be natural numbers. Suppose $n$ is sufficiently large for $C$ and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $j < \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(27) For every simple closed curve $C$ and for every natural number $n$ such that $n$ is sufficiently large for $C$ holds $\text{Y-SpanStart}(C,n) < \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(28) Let $C$ be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and $n$, $m$ be natural numbers. If $m \geq n$ and $n \geq 1$, then $\text{X-SpanStart}(C,m) = 2^{m -' n} \cdot (\text{X-SpanStart}(C,n) - 2) + 2$.

(29) Let $C$ be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and $n$, $m$ be natural numbers. Suppose $n \leq m$ and $n$ is sufficiently large for $C$. Then $m$ is sufficiently large for $C$.

(30) Let $G$ be a Go-board, $f$ be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and $i$, $j$ be natural numbers. Suppose $f$ is a sequence which elements belong to $G$ and special and $i \leq \mathrm{len}\,G$ and $j \leq \mathrm{width}\,G$. Then $\mathrm{cell}(G,i,j) \setminus \widetilde{\mathcal{L}}(f)$ is connected.

(31) Let $C$ be a simple closed curve and $n$, $k$ be natural numbers. Suppose $n$ is sufficiently large for $C$ and $\text{Y-SpanStart}(C,n) \leq k$ and $k \leq 2^{n -' \mathrm{ApproxIndex}\,C} \cdot (\text{Y-InitStart}\,C -' 2) + 2$. Then $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, k) \setminus \widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(32) Let $C$ be a subset of $\mathcal{E}^2_{\mathrm{T}}$ and $n$, $m$, $i$ be natural numbers. If $m \leq n$ and $1 < i$ and $i + 1 < \mathrm{len}\,\mathrm{Gauge}(C,m)$, then $2^{n -' m} \cdot (i - 2) + 2 + 1 < \mathrm{len}\,\mathrm{Gauge}(C,n)$.

(33) Let $C$ be a simple closed curve and $n$, $m$ be natural numbers. If $n$ is sufficiently large for $C$ and $n \leq m$, then $\mathrm{RightComp}(\mathrm{Span}(C,n))$ meets $\mathrm{RightComp}(\mathrm{Span}(C,m))$.

(34) Let $G$ be a Go-board and $f$ be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $f$ is a sequence which elements belong to $G$ and special. Let $i$, $j$ be natural numbers. If $i \leq \mathrm{len}\,G$ and $j \leq \mathrm{width}\,G$, then $\mathrm{Int}\,\mathrm{cell}(G,i,j) \subseteq (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(35) Let $C$ be a simple closed curve and $n$, $m$ be natural numbers. If $n$ is sufficiently large for $C$ and $n \leq m$, then $\widetilde{\mathcal{L}}(\mathrm{Span}(C,m)) \subseteq \overline{\mathrm{LeftComp}(\mathrm{Span}(C,n))}$.

(36) Let $C$ be a simple closed curve and $n$, $m$ be natural numbers. If $n$ is sufficiently large for $C$ and $n \leq m$, then $\mathrm{RightComp}(\mathrm{Span}(C,n)) \subseteq \mathrm{RightComp}(\mathrm{Span}(C,m))$.

(37) Let $C$ be a simple closed curve and $n$, $m$ be natural numbers. If $n$ is sufficiently large for $C$ and $n \leq m$, then $\mathrm{LeftComp}(\mathrm{Span}(C,m)) \subseteq \mathrm{LeftComp}(\mathrm{Span}(C,n))$.

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*Received June 27, 2002*

*Published January 2, 2004*