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Preparing the Internal Approximations of Simple Closed Curves[1]

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Summary. We mean by an internal approximation of a simple closed curve a special polygon disjoint with it but sufficiently close to it, i.e. such that it is clock-wise oriented and its right cells meet the curve. We prove lemmas used in the next article to construct a sequence of internal approximations.

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The articles [18], [6], [21], [2], [20], [12], [1], [16], [3], [22], [5], [4], [13], [17], [8], [9], [10], [11], [14], [15], [7], and [19] provide the notation and terminology for this paper.

In this paper j, k, n denote natural numbers and C denotes a subset of $\mathcal{E}^2_{\rm T}$ satisfying conditions of simple closed curve.

Let us consider C. The functor ApproxIndex$C$ yields a natural number and is defined as follows:

(Def. 1) ApproxIndex$C$ is sufficiently large for C and for every j such that j is sufficiently large for C holds $j \geq$ ApproxIndex$C$.

The following proposition is true

(1) $\mathrm{ApproxIndex}C \geq 1$.

Let us consider C. The functor Y-InitStart$C$ yields a natural number and is defined by:

(Def. 2) $\mathrm{Y\text{-}InitStart}C < \mathrm{width}\,\mathrm{Gauge}(C, \mathrm{ApproxIndex}C)$ and $\mathrm{cell}(\mathrm{Gauge}(C, \mathrm{ApproxIndex}C), \mathrm{X\text{-}SpanStart}(C, \mathrm{ApproxIndex}C)$ $1, \mathrm{Y\text{-}InitStart}C) \subseteq \mathrm{BDD}C$ and for every j such that $j < \mathrm{width}\,\mathrm{Gauge}(C, \mathrm{ApproxIndex}C)$ and $\mathrm{cell}(\mathrm{Gauge}(C, \mathrm{ApproxIndex}C), \mathrm{X\text{-}SpanStart}(C, \mathrm{ApproxIndex}C) -' 1, j) \subseteq \mathrm{BDD}C$ holds $j \geq \mathrm{Y\text{-}InitStart}C$.

One can prove the following propositions:

(2) $\mathrm{Y\text{-}InitStart}C > 1$.

(3) $\mathrm{Y\text{-}InitStart}C + 1 < \mathrm{width}\,\mathrm{Gauge}(C, \mathrm{ApproxIndex}C)$.

Let us consider C, n. Let us assume that n is sufficiently large for C. The functor Y-SpanStart(C,n) yields a natural number and is defined by the conditions (Def. 3).

(Def. 3)(i) $\mathrm{Y\text{-}SpanStart}(C,n) \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$,

(ii) for every k such that $\mathrm{Y\text{-}SpanStart}(C,n) \leq k$ and $k \leq 2^{n -' \mathrm{ApproxIndex}C} \cdot (\mathrm{Y\text{-}InitStart}C -' 2) + 2$ holds $\mathrm{cell}(\mathrm{Gauge}(C,n), \mathrm{X\text{-}SpanStart}(C,n) -' 1, k) \subseteq \mathrm{BDD}C$, and

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(iii) for every j such that $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and for every k such that $j \leq k$ and $k \leq 2^{n-'\mathrm{ApproxIndex}\,C} \cdot (\text{Y-InitStart}\,C -' 2) + 2$ holds $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, k) \subseteq \mathrm{BDD}\,C$ holds $j \geq \text{Y-SpanStart}(C,n)$.

The following propositions are true:

(4) If n is sufficiently large for C, then $\text{X-SpanStart}(C,n) = 2^{n-'\mathrm{ApproxIndex}\,C} \cdot (\text{X-SpanStart}(C, \mathrm{ApproxIndex}\,C) - 2) + 2$.

(5) If n is sufficiently large for C, then $\text{Y-SpanStart}(C,n) \leq 2^{n-'\mathrm{ApproxIndex}\,C} \cdot (\text{Y-InitStart}\,C -' 2) + 2$.

(6) If n is sufficiently large for C, then $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, \text{Y-SpanStart}(C,n)) \subseteq \mathrm{BDD}\,C$.

(7) If n is sufficiently large for C, then $1 < \text{Y-SpanStart}(C,n)$ and $\text{Y-SpanStart}(C,n) \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(8) If n is sufficiently large for C, then $\langle \text{X-SpanStart}(C,n), \text{Y-SpanStart}(C,n) \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$.

(9) If n is sufficiently large for C, then $\langle \text{X-SpanStart}(C,n) -' 1, \text{Y-SpanStart}(C,n) \rangle \in$ the indices of $\mathrm{Gauge}(C,n)$.

(10) If n is sufficiently large for C, then $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, \text{Y-SpanStart}(C,n) -' 1)$ meets C.

(11) If n is sufficiently large for C, then $\mathrm{cell}(\mathrm{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, \text{Y-SpanStart}(C,n))$ misses C.

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