JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Properties of the External Approximation of Jordan's Curve

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MML Identifier: JORDAN10.

WWW: http://mizar.org/JFM/Vol11/jordan10.html

The articles [20], [23], [18], [10], [1], [12], [2], [13], [22], [19], [17], [7], [3], [11], [8], [9], [15], [5], [21], [14], [16], [4], and [6] provide the notation and terminology for this paper.

Let us observe that there exists a subset of $\mathcal{E}^2_{\rm T}$ which is connected, compact, non vertical, and non horizontal.

We follow the rules: i, j, k, n denote natural numbers, P denotes a subset of $\mathcal{E}^2_{\rm T}$, and C denotes a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$.

One can prove the following propositions:

(1) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \operatorname{len} \operatorname{Cage}(C,n)$,

(iii) $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(iv) $\langle i, j+1 \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(v) $(\operatorname{Cage}(C,n))_k = \operatorname{Gauge}(C,n) \circ (i,j)$, and

(vi) $(\operatorname{Cage}(C,n))_{k+1} = \operatorname{Gauge}(C,n) \circ (i,j+1)$.

Then $i < \operatorname{len} \operatorname{Gauge}(C,n)$.

(2) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \operatorname{len} \operatorname{Cage}(C,n)$,

(iii) $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(iv) $\langle i, j+1 \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(v) $(\operatorname{Cage}(C,n))_k = \operatorname{Gauge}(C,n) \circ (i,j+1)$, and

(vi) $(\operatorname{Cage}(C,n))_{k+1} = \operatorname{Gauge}(C,n) \circ (i,j)$.

Then $i > 1$.

(3) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \operatorname{len} \operatorname{Cage}(C,n)$,

(iii) $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$,

(iv) $\langle i+1, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$,

(v) $(\mathrm{Cage}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i, j)$, and

(vi) $(\mathrm{Cage}(C,n))_{k+1} = \mathrm{Gauge}(C,n) \circ (i+1, j)$.

Then $j > 1$.

(4) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \mathrm{len}\,\mathrm{Cage}(C,n)$,

(iii) $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$,

(iv) $\langle i+1, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$,

(v) $(\mathrm{Cage}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i+1, j)$, and

(vi) $(\mathrm{Cage}(C,n))_{k+1} = \mathrm{Gauge}(C,n) \circ (i, j)$.

Then $j < \mathrm{width}\,\mathrm{Gauge}(C,n)$.

(5) C misses $\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$.

(6) $\text{N-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) = \text{N-bound}(C) + \frac{\text{N-bound}(C) - \text{S-bound}(C)}{2^n}$.

(7) If $i < j$, then $\text{N-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,j))) < \text{N-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,i)))$.

Let C be a connected compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and let n be a natural number. Note that $\overline{\mathrm{RightComp}(\mathrm{Cage}(C,n))}$ is compact.

We now state several propositions:

(8) $\mathrm{N}_{\mathrm{min}}(C) \in \mathrm{RightComp}(\mathrm{Cage}(C,n))$.

(9) C meets $\mathrm{RightComp}(\mathrm{Cage}(C,n))$.

(10) C misses $\mathrm{LeftComp}(\mathrm{Cage}(C,n))$.

(11) $C \subseteq \mathrm{RightComp}(\mathrm{Cage}(C,n))$.

(12) $C \subseteq \mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$.

(13) $\mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)) \subseteq \mathrm{UBD}\,C$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. The functor UBD-Family$C$ is defined as follows:

(Def. 1) $\text{UBD-Family}\,C = \{\mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)) : n$ ranges over natural numbers$\}$.

The functor BDD-Family$C$ is defined as follows:

(Def. 2) $\text{BDD-Family}\,C = \{\overline{\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))} : n$ ranges over natural numbers$\}$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. Then UBD-Family$C$ is a family of subsets of $\mathcal{E}^2_{\mathrm{T}}$. Then BDD-Family$C$ is a family of subsets of $\mathcal{E}^2_{\mathrm{T}}$.

Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. One can check that UBD-Family$C$ is non empty and BDD-Family$C$ is non empty.

The following propositions are true:

(14) $\bigcup \text{UBD-Family}\,C = \mathrm{UBD}\,C$.

(15) $C \subseteq \bigcap \text{BDD-Family}\,C$.

(16) $\mathrm{BDD}\,C$ misses $\mathrm{LeftComp}(\mathrm{Cage}(C,n))$.

(17) $\mathrm{BDD}\,C \subseteq \mathrm{RightComp}(\mathrm{Cage}(C,n))$.

(18) If P is inside component of C, then P misses $\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$.

(19) $\mathrm{BDD}C$ misses $\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$.

(20) $(\text{UBD-Family}\,C)^{\mathrm{c}} = \text{BDD-Family}\,C$.

(21) $\bigcap \text{BDD-Family}\,C = C \cup \mathrm{BDD}C$.

ACKNOWLEDGMENTS

I would like to thank Professor Yatsuka Nakamura for his help in the preparation of the article.

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Received June 24, 1999

Published January 2, 2004