

On Outside Fashoda Meet Theorem

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Summary. We have proven the “Fashoda Meet Theorem” in [14]. Here we prove the outside version of it. It says that if Britain and France intended to set the courses for ships to the opposite side of Africa, they must also meet.

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The articles [18], [21], [1], [19], [12], [22], [4], [2], [7], [6], [16], [11], [3], [13], [5], [20], [9], [10], [15], [8], and [17] provide the notation and terminology for this paper.

One can prove the following propositions:

- (2)¹ For every real number a such that $1 \leq a$ holds $a \leq a^2$.
- (3) For every real number a such that $-1 \geq a$ holds $-a \leq a^2$.
- (4) For every real number a such that $-1 > a$ holds $-a < a^2$.
- (5) For all real numbers a, b such that $b^2 \leq a^2$ and $a \geq 0$ holds $-a \leq b$ and $b \leq a$.
- (6) For all real numbers a, b such that $b^2 < a^2$ and $a \geq 0$ holds $-a < b$ and $b < a$.
- (7) For all real numbers a, b such that $-a \leq b$ and $b \leq a$ holds $b^2 \leq a^2$.
- (8) For all real numbers a, b such that $-a < b$ and $b < a$ holds $b^2 < a^2$.

In the sequel T, T_1, T_2, S denote non empty topological spaces.

Next we state a number of propositions:

- (9) Let f be a map from T_1 into S , g be a map from T_2 into S , and F_1, F_2 be subsets of T . Suppose that T_1 is a subspace of T and T_2 is a subspace of T and $F_1 = \Omega_{(T_1)}$ and $F_2 = \Omega_{(T_2)}$ and $\Omega_{(T_1)} \cup \Omega_{(T_2)} = \Omega_T$ and F_1 is closed and F_2 is closed and f is continuous and g is continuous and for every set p such that $p \in \Omega_{(T_1)} \cap \Omega_{(T_2)}$ holds $f(p) = g(p)$. Then there exists a map h from T into S such that $h = f + g$ and h is continuous.
- (10) Let n be a natural number, q_2 be a point of \mathcal{E}^n , q be a point of \mathcal{E}_T^n , and r be a real number. If $q = q_2$, then $\text{Ball}(q_2, r) = \{q_3; q_3 \text{ ranges over points of } \mathcal{E}_T^n: |q - q_3| < r\}$.
- (11) $(0_{\mathcal{E}_T^2})_1 = 0$ and $(0_{\mathcal{E}_T^2})_2 = 0$.
- (12) $1.\text{REAL}2 = \langle 1, 1 \rangle$.
- (13) $(1.\text{REAL}2)_1 = 1$ and $(1.\text{REAL}2)_2 = 1$.

¹ The proposition (1) has been removed.

- (14) $\text{dom proj1} = \text{the carrier of } \mathcal{E}_T^2 \text{ and } \text{dom proj1} = \mathcal{R}^2.$
- (15) $\text{dom proj2} = \text{the carrier of } \mathcal{E}_T^2 \text{ and } \text{dom proj2} = \mathcal{R}^2.$
- (16) proj1 is a map from \mathcal{E}_T^2 into $\mathbb{R}^1.$
- (17) proj2 is a map from \mathcal{E}_T^2 into $\mathbb{R}^1.$
- (18) For every point p of \mathcal{E}_T^2 holds $p = [\text{proj1}(p), \text{proj2}(p)].$
- (19) For every subset B of \mathcal{E}_T^2 such that $B = \{0_{\mathcal{E}_T^2}\}$ holds $B^c \neq \emptyset$ and $(\text{the carrier of } \mathcal{E}_T^2) \setminus B \neq \emptyset.$
- (20) Let X, Y be non empty topological spaces and f be a map from X into Y . Then f is continuous if and only if for every point p of X and for every subset V of Y such that $f(p) \in V$ and V is open there exists a subset W of X such that $p \in W$ and W is open and $f^c W \subseteq V.$
- (21) Let p be a point of \mathcal{E}_T^2 and G be a subset of \mathcal{E}_T^2 . Suppose G is open and $p \in G$. Then there exists a real number r such that $r > 0$ and $\{q; q \text{ ranges over points of } \mathcal{E}_T^2: p_1 - r < q_1 \wedge q_1 < p_1 + r \wedge p_2 - r < q_2 \wedge q_2 < p_2 + r\} \subseteq G.$
- (22) Let X, Y, Z be non empty topological spaces, B be a subset of Y , C be a subset of Z , f be a map from X into Y , and h be a map from $Y \upharpoonright B$ into $Z \upharpoonright C$. Suppose f is continuous and h is continuous and $\text{rng } f \subseteq B$ and $B \neq \emptyset$ and $C \neq \emptyset$. Then there exists a map g from X into Z such that g is continuous and $g = h \cdot f.$

In the sequel p, q are points of $\mathcal{E}_T^2.$

The function OutInSq from $(\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ into $(\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ is defined by the condition (Def. 1).

(Def. 1) Let p be a point of \mathcal{E}_T^2 such that $p \neq 0_{\mathcal{E}_T^2}$. Then

- (i) if $p_2 \leq p_1$ and $-p_1 \leq p_2$ or $p_2 \geq p_1$ and $p_2 \leq -p_1$, then $\text{OutInSq}(p) = [\frac{1}{p_1}, \frac{p_2}{p_1}]$, and
- (ii) if $p_2 \not\leq p_1$ or $-p_1 \not\leq p_2$ and if $p_2 \not\geq p_1$ or $p_2 \not\leq -p_1$, then $\text{OutInSq}(p) = [\frac{p_2}{p_2}, \frac{1}{p_2}]$.

The following propositions are true:

- (23) Let p be a point of \mathcal{E}_T^2 . Suppose $p_2 \not\leq p_1$ or $-p_1 \not\leq p_2$ but $p_2 \not\geq p_1$ or $p_2 \not\leq -p_1$. Then $p_1 \leq p_2$ and $-p_2 \leq p_1$ or $p_1 \geq p_2$ and $p_1 \leq -p_2$.
- (24) Let p be a point of \mathcal{E}_T^2 such that $p \neq 0_{\mathcal{E}_T^2}$. Then
- (i) if $p_1 \leq p_2$ and $-p_2 \leq p_1$ or $p_1 \geq p_2$ and $p_1 \leq -p_2$, then $\text{OutInSq}(p) = [\frac{p_2}{p_2}, \frac{1}{p_2}]$, and
- (ii) if $p_1 \not\leq p_2$ or $-p_2 \not\leq p_1$ and if $p_1 \not\geq p_2$ or $p_1 \not\leq -p_2$, then $\text{OutInSq}(p) = [\frac{1}{p_1}, \frac{p_2}{p_1}]$.
- (25) Let D be a subset of \mathcal{E}_T^2 and K_0 be a subset of $(\mathcal{E}_T^2) \upharpoonright D$. Suppose $K_0 = \{p : (p_2 \leq p_1 \wedge -p_1 \leq p_2 \vee p_2 \geq p_1 \wedge p_2 \leq -p_1) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then $\text{rng}(\text{OutInSq} \upharpoonright K_0) \subseteq \text{the carrier of } (\mathcal{E}_T^2) \upharpoonright D \upharpoonright K_0.$
- (26) Let D be a subset of \mathcal{E}_T^2 and K_0 be a subset of $(\mathcal{E}_T^2) \upharpoonright D$. Suppose $K_0 = \{p : (p_1 \leq p_2 \wedge -p_2 \leq p_1 \vee p_1 \geq p_2 \wedge p_1 \leq -p_2) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then $\text{rng}(\text{OutInSq} \upharpoonright K_0) \subseteq \text{the carrier of } (\mathcal{E}_T^2) \upharpoonright D \upharpoonright K_0.$
- (27) Let K_1 be a set and D be a non empty subset of \mathcal{E}_T^2 . Suppose $K_1 = \{p; p \text{ ranges over points of } \mathcal{E}_T^2: (p_2 \leq p_1 \wedge -p_1 \leq p_2 \vee p_2 \geq p_1 \wedge p_2 \leq -p_1) \wedge p \neq 0_{\mathcal{E}_T^2}\}$ and $D^c = \{0_{\mathcal{E}_T^2}\}$. Then K_1 is a non empty subset of $(\mathcal{E}_T^2) \upharpoonright D$ and a non empty subset of $\mathcal{E}_T^2.$
- (28) Let K_1 be a set and D be a non empty subset of \mathcal{E}_T^2 . Suppose $K_1 = \{p; p \text{ ranges over points of } \mathcal{E}_T^2: (p_1 \leq p_2 \wedge -p_2 \leq p_1 \vee p_1 \geq p_2 \wedge p_1 \leq -p_2) \wedge p \neq 0_{\mathcal{E}_T^2}\}$ and $D^c = \{0_{\mathcal{E}_T^2}\}$. Then K_1 is a non empty subset of $(\mathcal{E}_T^2) \upharpoonright D$ and a non empty subset of $\mathcal{E}_T^2.$

- (29) Let X be a non empty topological space and f_1, f_2 be maps from X into \mathbb{R}^1 . Suppose f_1 is continuous and f_2 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for all real numbers r_1, r_2 such that $f_1(p) = r_1$ and $f_2(p) = r_2$ holds $g(p) = r_1 + r_2$ and g is continuous.
- (30) Let X be a non empty topological space and a be a real number. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X holds $g(p) = a$ and g is continuous.
- (31) Let X be a non empty topological space and f_1, f_2 be maps from X into \mathbb{R}^1 . Suppose f_1 is continuous and f_2 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for all real numbers r_1, r_2 such that $f_1(p) = r_1$ and $f_2(p) = r_2$ holds $g(p) = r_1 - r_2$ and g is continuous.
- (32) Let X be a non empty topological space and f_1 be a map from X into \mathbb{R}^1 . Suppose f_1 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for every real number r_1 such that $f_1(p) = r_1$ holds $g(p) = r_1 \cdot r_1$ and g is continuous.
- (33) Let X be a non empty topological space, f_1 be a map from X into \mathbb{R}^1 , and a be a real number. Suppose f_1 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for every real number r_1 such that $f_1(p) = r_1$ holds $g(p) = a \cdot r_1$ and g is continuous.
- (34) Let X be a non empty topological space, f_1 be a map from X into \mathbb{R}^1 , and a be a real number. Suppose f_1 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for every real number r_1 such that $f_1(p) = r_1$ holds $g(p) = r_1 + a$ and g is continuous.
- (35) Let X be a non empty topological space and f_1, f_2 be maps from X into \mathbb{R}^1 . Suppose f_1 is continuous and f_2 is continuous. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for all real numbers r_1, r_2 such that $f_1(p) = r_1$ and $f_2(p) = r_2$ holds $g(p) = r_1 \cdot r_2$ and g is continuous.
- (36) Let X be a non empty topological space and f_1 be a map from X into \mathbb{R}^1 . Suppose f_1 is continuous and for every point q of X holds $f_1(q) \neq 0$. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for every real number r_1 such that $f_1(p) = r_1$ holds $g(p) = \frac{1}{r_1}$ and g is continuous.
- (37) Let X be a non empty topological space and f_1, f_2 be maps from X into \mathbb{R}^1 . Suppose f_1 is continuous and f_2 is continuous and for every point q of X holds $f_2(q) \neq 0$. Then there exists a map g from X into \mathbb{R}^1 such that for every point p of X and for all real numbers r_1, r_2 such that $f_1(p) = r_1$ and $f_2(p) = r_2$ holds $g(p) = \frac{r_1}{r_2}$ and g is continuous.
- (38) Let X be a non empty topological space and f_1, f_2 be maps from X into \mathbb{R}^1 . Suppose f_1 is continuous and f_2 is continuous and for every point q of X holds $f_2(q) \neq 0$. Then there exists a map g from X into \mathbb{R}^1 such that
- (i) for every point p of X and for all real numbers r_1, r_2 such that $f_1(p) = r_1$ and $f_2(p) = r_2$ holds $g(p) = \frac{r_1}{r_2}$, and
 - (ii) g is continuous.
- (39) Let K_0 be a subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2) \upharpoonright K_0$ into \mathbb{R}^1 . If for every point p of $(\mathcal{E}_T^2) \upharpoonright K_0$ holds $f(p) = \text{proj}1(p)$, then f is continuous.
- (40) Let K_0 be a subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2) \upharpoonright K_0$ into \mathbb{R}^1 . If for every point p of $(\mathcal{E}_T^2) \upharpoonright K_0$ holds $f(p) = \text{proj}2(p)$, then f is continuous.
- (41) Let K_2 be a non empty subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2) \upharpoonright K_2$ into \mathbb{R}^1 . Suppose that
- (i) for every point p of \mathcal{E}_T^2 such that $p \in$ the carrier of $(\mathcal{E}_T^2) \upharpoonright K_2$ holds $f(p) = \frac{1}{p_1}$, and
 - (ii) for every point q of \mathcal{E}_T^2 such that $q \in$ the carrier of $(\mathcal{E}_T^2) \upharpoonright K_2$ holds $q_1 \neq 0$.
- Then f is continuous.

(42) Let K_2 be a non empty subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2)|_{K_2}$ into \mathbb{R}^1 . Suppose that

- (i) for every point p of \mathcal{E}_T^2 such that $p \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $f(p) = \frac{1}{p_2}$, and
- (ii) for every point q of \mathcal{E}_T^2 such that $q \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $q_2 \neq 0$.

Then f is continuous.

(43) Let K_2 be a non empty subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2)|_{K_2}$ into \mathbb{R}^1 . Suppose that

- (i) for every point p of \mathcal{E}_T^2 such that $p \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $f(p) = \frac{p_2}{p_1}$, and
- (ii) for every point q of \mathcal{E}_T^2 such that $q \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $q_1 \neq 0$.

Then f is continuous.

(44) Let K_2 be a non empty subset of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2)|_{K_2}$ into \mathbb{R}^1 . Suppose that

- (i) for every point p of \mathcal{E}_T^2 such that $p \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $f(p) = \frac{p_1}{p_2}$, and
- (ii) for every point q of \mathcal{E}_T^2 such that $q \in$ the carrier of $(\mathcal{E}_T^2)|_{K_2}$ holds $q_2 \neq 0$.

Then f is continuous.

(45) Let K_0, B_0 be subsets of \mathcal{E}_T^2 , f be a map from $(\mathcal{E}_T^2)|_{K_0}$ into $(\mathcal{E}_T^2)|_{B_0}$, and f_1, f_2 be maps from $(\mathcal{E}_T^2)|_{K_0}$ into \mathbb{R}^1 . Suppose that

- (i) f_1 is continuous,
- (ii) f_2 is continuous,
- (iii) $K_0 \neq \emptyset$,
- (iv) $B_0 \neq \emptyset$, and
- (v) for all real numbers x, y, r, s such that $[x, y] \in K_0$ and $r = f_1([x, y])$ and $s = f_2([x, y])$ holds $f([x, y]) = [r, s]$.

Then f is continuous.

(46) Let K_0, B_0 be subsets of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2)|_{K_0}$ into $(\mathcal{E}_T^2)|_{B_0}$. Suppose $f = \text{OutInSq}|_{K_0}$ and $B_0 = (\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ and $K_0 = \{p : (p_2 \leq p_1 \wedge -p_1 \leq p_2 \vee p_2 \geq p_1 \wedge p_2 \leq -p_1) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then f is continuous.

(47) Let K_0, B_0 be subsets of \mathcal{E}_T^2 and f be a map from $(\mathcal{E}_T^2)|_{K_0}$ into $(\mathcal{E}_T^2)|_{B_0}$. Suppose $f = \text{OutInSq}|_{K_0}$ and $B_0 = (\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ and $K_0 = \{p : (p_1 \leq p_2 \wedge -p_2 \leq p_1 \vee p_1 \geq p_2 \wedge p_1 \leq -p_2) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then f is continuous.

In this article we present several logical schemes. The scheme *TopSubset* concerns a unary predicate \mathcal{P} , and states that:

$$\{p; p \text{ ranges over points of } \mathcal{E}_T^2: \mathcal{P}[p]\} \text{ is a subset of } \mathcal{E}_T^2$$

for all values of the parameters.

The scheme *TopCompl* deals with a subset \mathcal{A} of \mathcal{E}_T^2 and a unary predicate \mathcal{P} , and states that:

$$\mathcal{A}^c = \{p; p \text{ ranges over points of } \mathcal{E}_T^2: \text{not } \mathcal{P}[p]\}$$

provided the parameters satisfy the following condition:

- $\mathcal{A} = \{p; p \text{ ranges over points of } \mathcal{E}_T^2: \mathcal{P}[p]\}$.

The scheme *ClosedSubset* deals with two unary functors \mathcal{F} and \mathcal{G} yielding real numbers, and states that:

$$\{p; p \text{ ranges over points of } \mathcal{E}_T^2: \mathcal{F}(p) \leq \mathcal{G}(p)\} \text{ is a closed subset of } \mathcal{E}_T^2$$

provided the parameter meets the following requirements:

- For all points p, q of \mathcal{E}_T^2 holds $\mathcal{F}(p - q) = \mathcal{F}(p) - \mathcal{F}(q)$ and $\mathcal{G}(p - q) = \mathcal{G}(p) - \mathcal{G}(q)$, and
- For all points p, q of \mathcal{E}_T^2 holds $|p - q|^2 = \mathcal{F}(p - q)^2 + \mathcal{G}(p - q)^2$.

Next we state several propositions:

- (48) Let B_0 be a subset of \mathcal{E}_T^2 , K_0 be a subset of $(\mathcal{E}_T^2)|B_0$, and f be a map from $(\mathcal{E}_T^2)|B_0|K_0$ into $(\mathcal{E}_T^2)|B_0$. Suppose $f = \text{OutInSq}|K_0$ and $B_0 = (\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ and $K_0 = \{p : (p_2 \leq p_1 \wedge -p_1 \leq p_2 \vee p_2 \geq p_1 \wedge p_2 \leq -p_1) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then f is continuous and K_0 is closed.
- (49) Let B_0 be a subset of \mathcal{E}_T^2 , K_0 be a subset of $(\mathcal{E}_T^2)|B_0$, and f be a map from $(\mathcal{E}_T^2)|B_0|K_0$ into $(\mathcal{E}_T^2)|B_0$. Suppose $f = \text{OutInSq}|K_0$ and $B_0 = (\text{the carrier of } \mathcal{E}_T^2) \setminus \{0_{\mathcal{E}_T^2}\}$ and $K_0 = \{p : (p_1 \leq p_2 \wedge -p_2 \leq p_1 \vee p_1 \geq p_2 \wedge p_1 \leq -p_2) \wedge p \neq 0_{\mathcal{E}_T^2}\}$. Then f is continuous and K_0 is closed.
- (50) Let D be a non empty subset of \mathcal{E}_T^2 . Suppose $D^c = \{0_{\mathcal{E}_T^2}\}$. Then there exists a map h from $(\mathcal{E}_T^2)|D$ into $(\mathcal{E}_T^2)|D$ such that $h = \text{OutInSq}$ and h is continuous.
- (51) Let B, K_0, K_3 be subsets of \mathcal{E}_T^2 . Suppose that
- (i) $B = \{0_{\mathcal{E}_T^2}\}$,
 - (ii) $K_0 = \{p : -1 < p_1 \wedge p_1 < 1 \wedge -1 < p_2 \wedge p_2 < 1\}$, and
 - (iii) $K_3 = \{q : -1 = q_1 \wedge -1 \leq q_2 \wedge q_2 \leq 1 \vee q_1 = 1 \wedge -1 \leq q_2 \wedge q_2 \leq 1 \vee -1 = q_2 \wedge -1 \leq q_1 \wedge q_1 \leq 1 \vee 1 = q_2 \wedge -1 \leq q_1 \wedge q_1 \leq 1\}$.
- Then there exists a map f from $(\mathcal{E}_T^2)|B^c$ into $(\mathcal{E}_T^2)|B^c$ such that
- (iv) f is continuous and one-to-one,
 - (v) for every point t of \mathcal{E}_T^2 such that $t \in K_0$ and $t \neq 0_{\mathcal{E}_T^2}$ holds $f(t) \notin K_0 \cup K_3$,
 - (vi) for every point r of \mathcal{E}_T^2 such that $r \notin K_0 \cup K_3$ holds $f(r) \in K_0$, and
 - (vii) for every point s of \mathcal{E}_T^2 such that $s \in K_3$ holds $f(s) = s$.
- (52) Let f, g be maps from \mathbb{I} into \mathcal{E}_T^2 , K_0 be a subset of \mathcal{E}_T^2 , and O, I be points of \mathbb{I} . Suppose that $O = 0$ and $I = 1$ and f is continuous and one-to-one and g is continuous and one-to-one and $K_0 = \{p : -1 < p_1 \wedge p_1 < 1 \wedge -1 < p_2 \wedge p_2 < 1\}$ and $f(O)_1 = -1$ and $f(I)_1 = 1$ and $-1 \leq f(O)_2$ and $f(O)_2 \leq 1$ and $-1 \leq f(I)_2$ and $f(I)_2 \leq 1$ and $g(O)_2 = -1$ and $g(I)_2 = 1$ and $-1 \leq g(O)_1$ and $g(O)_1 \leq 1$ and $-1 \leq g(I)_1$ and $g(I)_1 \leq 1$ and $\text{rng } f$ misses K_0 and $\text{rng } g$ misses K_0 . Then $\text{rng } f$ meets $\text{rng } g$.
- (53) Let A, B, C, D be real numbers and f be a map from \mathcal{E}_T^2 into \mathcal{E}_T^2 . Suppose that for every point t of \mathcal{E}_T^2 holds $f(t) = [A \cdot t_1 + B, C \cdot t_2 + D]$. Then f is continuous.

Let A, B, C, D be real numbers. The functor $\text{AffineMap}(A, B, C, D)$ yields a map from \mathcal{E}_T^2 into \mathcal{E}_T^2 and is defined as follows:

(Def. 2) For every point t of \mathcal{E}_T^2 holds $(\text{AffineMap}(A, B, C, D))(t) = [A \cdot t_1 + B, C \cdot t_2 + D]$.

Let a, b, c, d be real numbers. One can check that $\text{AffineMap}(a, b, c, d)$ is continuous.

Next we state several propositions:

- (54) For all real numbers A, B, C, D such that $A > 0$ and $C > 0$ holds $\text{AffineMap}(A, B, C, D)$ is one-to-one.
- (55) Let f, g be maps from \mathbb{I} into \mathcal{E}_T^2 , a, b, c, d be real numbers, and O, I be points of \mathbb{I} . Suppose that $O = 0$ and $I = 1$ and f is continuous and one-to-one and g is continuous and one-to-one and $f(O)_1 = a$ and $f(I)_1 = b$ and $c \leq f(O)_2$ and $f(O)_2 \leq d$ and $c \leq f(I)_2$ and $f(I)_2 \leq d$ and $g(O)_2 = c$ and $g(I)_2 = d$ and $a \leq g(O)_1$ and $g(O)_1 \leq b$ and $a \leq g(I)_1$ and $g(I)_1 \leq b$ and $a < b$ and $c < d$ and it is not true that there exists a point r of \mathbb{I} such that $a < f(r)_1$ and $f(r)_1 < b$ and $c < f(r)_2$ and $f(r)_2 < d$ and it is not true that there exists a point r of \mathbb{I} such that $a < g(r)_1$ and $g(r)_1 < b$ and $c < g(r)_2$ and $g(r)_2 < d$. Then $\text{rng } f$ meets $\text{rng } g$.
- (56)(i) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: (p_7)_2 \leq (p_7)_1\}$ is a closed subset of \mathcal{E}_T^2 , and
- (ii) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: (p_7)_1 \leq (p_7)_2\}$ is a closed subset of \mathcal{E}_T^2 .

- (57)(i) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: -(p_7)_1 \leq (p_7)_2\}$ is a closed subset of \mathcal{E}_T^2 , and
(ii) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: (p_7)_2 \leq -(p_7)_1\}$ is a closed subset of \mathcal{E}_T^2 .
- (58)(i) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: -(p_7)_2 \leq (p_7)_1\}$ is a closed subset of \mathcal{E}_T^2 , and
(ii) $\{p_7; p_7 \text{ ranges over points of } \mathcal{E}_T^2: (p_7)_1 \leq -(p_7)_2\}$ is a closed subset of \mathcal{E}_T^2 .

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