

Institution of Many Sorted Algebras. Part I: Signature Reduct of an Algebra

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Summary. In the paper the notation necessary to construct the institution of many sorted algebras is introduced.

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The articles [21], [14], [24], [25], [11], [8], [15], [1], [16], [2], [3], [9], [6], [22], [23], [13], [12], [10], [19], [18], [5], [20], [4], [17], and [7] provide the notation and terminology for this paper.

1. PRELIMINARIES

The following proposition is true

- (2)¹ Let S be a non empty non void many sorted signature, o be an operation symbol of S , V be a non-empty many sorted set indexed by the carrier of S , and x be a set. Then x is an argument sequence of $\text{Sym}(o, V)$ if and only if x is an element of $\text{Args}(o, \text{Free}(V))$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S , and let o be an operation symbol of S . One can verify that every element of $\text{Args}(o, \text{Free}(V))$ is decorated tree yielding.

The following two propositions are true:

- (3) Let S be a non empty non void many sorted signature and A_1, A_2 be algebras over S . Suppose the sorts of A_1 are transformable to the sorts of A_2 . Let o be an operation symbol of S . If $\text{Args}(o, A_1) \neq \emptyset$, then $\text{Args}(o, A_2) \neq \emptyset$.
- (4) Let S be a non empty non void many sorted signature, o be an operation symbol of S , V be a non-empty many sorted set indexed by the carrier of S , and x be an element of $\text{Args}(o, \text{Free}(V))$. Then $(\text{Den}(o, \text{Free}(V)))(x) = \langle o, \text{the carrier of } S \rangle\text{-tree}(x)$.

Let S be a non empty non void many sorted signature and let A be a non-empty algebra over S . Observe that the algebra of A is non-empty.

One can prove the following propositions:

- (5) Let S be a non empty non void many sorted signature and A, B be algebras over S . Suppose the algebra of $A =$ the algebra of B . Let o be an operation symbol of S . Then $\text{Den}(o, A) = \text{Den}(o, B)$.

¹ The proposition (1) has been removed.

(6) Let S be a non empty non void many sorted signature and A_1, A_2, B_1, B_2 be algebras over S . Suppose the algebra of $A_1 =$ the algebra of B_1 and the algebra of $A_2 =$ the algebra of B_2 . Let f be a many sorted function from A_1 into A_2 and g be a many sorted function from B_1 into B_2 . Suppose $f = g$. Let o be an operation symbol of S . Suppose $\text{Args}(o, A_1) \neq \emptyset$ and $\text{Args}(o, A_2) \neq \emptyset$. Let x be an element of $\text{Args}(o, A_1)$ and y be an element of $\text{Args}(o, B_1)$. If $x = y$, then $f\#x = g\#y$.

(7) Let S be a non empty non void many sorted signature and A_1, A_2, B_1, B_2 be algebras over S . Suppose that

- (i) the algebra of $A_1 =$ the algebra of B_1 ,
- (ii) the algebra of $A_2 =$ the algebra of B_2 , and
- (iii) the sorts of A_1 are transformable to the sorts of A_2 .

Let h be a many sorted function from A_1 into A_2 . Suppose h is a homomorphism of A_1 into A_2 . Then there exists a many sorted function h' from B_1 into B_2 such that $h' = h$ and h' is a homomorphism of B_1 into B_2 .

Let S be a many sorted signature. We say that S is feasible if and only if:

(Def. 1) If the carrier of $S = \emptyset$, then the operation symbols of $S = \emptyset$.

Next we state the proposition

(8) Let S be a many sorted signature. Then S is feasible if and only if $\text{dom}(\text{the result sort of } S) = \text{the operation symbols of } S$.

One can verify the following observations:

- * every many sorted signature which is non empty is also feasible,
- * every many sorted signature which is void is also feasible,
- * every many sorted signature which is empty and feasible is also void, and
- * every many sorted signature which is non void and feasible is also non empty.

Let us mention that there exists a many sorted signature which is non void and non empty.

Next we state two propositions:

(9) Let S be a feasible many sorted signature. Then $\text{id}_{\text{the carrier of } S}$ and $\text{id}_{\text{the operation symbols of } S}$ form morphism between S and S .

(10) Let S_1, S_2 be many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2 . Then

- (i) f is a function from the carrier of S_1 into the carrier of S_2 , and
- (ii) g is a function from the operation symbols of S_1 into the operation symbols of S_2 .

2. SUBSIGNATURES

Let S be a feasible many sorted signature. A many sorted signature is said to be a subsignature of S if:

(Def. 2) $\text{id}_{\text{the carrier of it}}$ and $\text{id}_{\text{the operation symbols of it}}$ form morphism between it and S .

The following proposition is true

(11) Let S be a feasible many sorted signature and T be a subsignature of S . Then the carrier of $T \subseteq$ the carrier of S and the operation symbols of $T \subseteq$ the operation symbols of S .

Let S be a feasible many sorted signature. Note that every subsignature of S is feasible. One can prove the following propositions:

- (12) Let S be a feasible many sorted signature and T be a subsignature of S . Then the result sort of $T \subseteq$ the result sort of S and the arity of $T \subseteq$ the arity of S .
- (13) Let S be a feasible many sorted signature and T be a subsignature of S . Then
- (i) the arity of $T = (\text{the arity of } S) \setminus (\text{the operation symbols of } T)$, and
 - (ii) the result sort of $T = (\text{the result sort of } S) \setminus (\text{the operation symbols of } T)$.
- (14) Let S, T be feasible many sorted signatures. Suppose that
- (i) the carrier of $T \subseteq$ the carrier of S ,
 - (ii) the arity of $T \subseteq$ the arity of S , and
 - (iii) the result sort of $T \subseteq$ the result sort of S .
- Then T is a subsignature of S .
- (15) Let S, T be feasible many sorted signatures. Suppose that
- (i) the carrier of $T \subseteq$ the carrier of S ,
 - (ii) the arity of $T = (\text{the arity of } S) \setminus (\text{the operation symbols of } T)$, and
 - (iii) the result sort of $T = (\text{the result sort of } S) \setminus (\text{the operation symbols of } T)$.
- Then T is a subsignature of S .
- (16) Every feasible many sorted signature S is a subsignature of S .
- (17) For every feasible many sorted signature S_1 and for every subsignature S_2 of S_1 holds every subsignature of S_2 is a subsignature of S_1 .
- (18) Let S_1 be a feasible many sorted signature and S_2 be a subsignature of S_1 . Suppose S_1 is a subsignature of S_2 . Then the many sorted signature of $S_1 =$ the many sorted signature of S_2 .

Let S be a non empty many sorted signature. Observe that there exists a subsignature of S which is non empty.

Let S be a non void feasible many sorted signature. One can check that there exists a subsignature of S which is non void.

We now state three propositions:

- (19) Let S be a feasible many sorted signature, S' be a subsignature of S , T be a many sorted signature, and f, g be functions. Suppose f and g form morphism between S and T . Then $f \setminus \text{the carrier of } S'$ and $g \setminus \text{the operation symbols of } S'$ form morphism between S' and T .
- (20) Let S be a many sorted signature, T be a feasible many sorted signature, T' be a subsignature of T , and f, g be functions. Suppose f and g form morphism between S and T' . Then f and g form morphism between S and T .
- (21) Let S be a many sorted signature, T be a feasible many sorted signature, T' be a subsignature of T , and f, g be functions. Suppose f and g form morphism between S and T and $\text{rng } f \subseteq \text{the carrier of } T'$ and $\text{rng } g \subseteq \text{the operation symbols of } T'$. Then f and g form morphism between S and T' .

3. SIGNATURE REDUCTS

Let S_1, S_2 be non empty many sorted signatures, let A be an algebra over S_2 , and let f, g be functions. Let us assume that f and g form morphism between S_1 and S_2 . The functor $A \setminus_{(f,g)} S_1$ yields a strict algebra over S_1 and is defined by the conditions (Def. 3).

- (Def. 3)(i) The sorts of $A \upharpoonright_{(f,g)} S_1 = (\text{the sorts of } A) \cdot f$, and
 (ii) the characteristics of $A \upharpoonright_{(f,g)} S_1 = (\text{the characteristics of } A) \cdot g$.

Let S_2, S_1 be non empty many sorted signatures and let A be an algebra over S_2 . The functor $A \upharpoonright S_1$ yielding a strict algebra over S_1 is defined as follows:

- (Def. 4) $A \upharpoonright S_1 = A \upharpoonright_{(\text{id}_{\text{the carrier of } S_1}, \text{id}_{\text{the operation symbols of } S_1})} S_1$.

We now state two propositions:

- (22) Let S_1, S_2 be non empty many sorted signatures and A, B be algebras over S_2 . Suppose the algebra of $A = \text{the algebra of } B$. Let f, g be functions. If f and g form morphism between S_1 and S_2 , then $A \upharpoonright_{(f,g)} S_1 = B \upharpoonright_{(f,g)} S_1$.
- (23) Let S_1, S_2 be non empty many sorted signatures, A be a non-empty algebra over S_2 , and f, g be functions. If f and g form morphism between S_1 and S_2 , then $A \upharpoonright_{(f,g)} S_1$ is non-empty.

Let S_2 be a non empty many sorted signature, let S_1 be a non empty subsignature of S_2 , and let A be a non-empty algebra over S_2 . Observe that $A \upharpoonright S_1$ is non-empty.

One can prove the following propositions:

- (24) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2 . Let A be an algebra over S_2 , o_1 be an operation symbol of S_1 , and o_2 be an operation symbol of S_2 . If $o_2 = g(o_1)$, then $\text{Den}(o_1, A \upharpoonright_{(f,g)} S_1) = \text{Den}(o_2, A)$.
- (25) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2 . Let A be an algebra over S_2 , o_1 be an operation symbol of S_1 , and o_2 be an operation symbol of S_2 . If $o_2 = g(o_1)$, then $\text{Args}(o_2, A) = \text{Args}(o_1, A \upharpoonright_{(f,g)} S_1)$ and $\text{Result}(o_1, A \upharpoonright_{(f,g)} S_1) = \text{Result}(o_2, A)$.
- (26) Let S be a non empty many sorted signature and A be an algebra over S . Then $A \upharpoonright_{(\text{id}_{\text{the carrier of } S}, \text{id}_{\text{the operation symbols of } S})} S = \text{the algebra of } A$.
- (27) For every non empty many sorted signature S and for every algebra A over S holds $A \upharpoonright S = \text{the algebra of } A$.
- (28) Let S_1, S_2, S_3 be non empty many sorted signatures and f_1, g_1 be functions. Suppose f_1 and g_1 form morphism between S_1 and S_2 . Let f_2, g_2 be functions. Suppose f_2 and g_2 form morphism between S_2 and S_3 . Let A be an algebra over S_3 . Then $A \upharpoonright_{(f_2 \cdot f_1, g_2 \cdot g_1)} S_1 = A \upharpoonright_{(f_2, g_2)} S_2 \upharpoonright_{(f_1, g_1)} S_1$.
- (29) Let S_1 be a non empty feasible many sorted signature, S_2 be a non empty subsignature of S_1 , S_3 be a non empty subsignature of S_2 , and A be an algebra over S_1 . Then $A \upharpoonright S_3 = A \upharpoonright S_2 \upharpoonright S_3$.
- (30) Let S_1, S_2 be non empty many sorted signatures, f be a function from the carrier of S_1 into the carrier of S_2 , and g be a function. Suppose f and g form morphism between S_1 and S_2 . Let A_1, A_2 be algebras over S_2 and h be a many sorted function from the sorts of A_1 into the sorts of A_2 . Then $h \cdot f$ is a many sorted function from the sorts of $A_1 \upharpoonright_{(f,g)} S_1$ into the sorts of $A_2 \upharpoonright_{(f,g)} S_1$.
- (31) Let S_1 be a non empty many sorted signature, S_2 be a non empty subsignature of S_1 , A_1, A_2 be algebras over S_1 , and h be a many sorted function from the sorts of A_1 into the sorts of A_2 . Then $h \upharpoonright_{\text{the carrier of } S_2}$ is a many sorted function from the sorts of $A_1 \upharpoonright S_2$ into the sorts of $A_2 \upharpoonright S_2$.
- (32) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2 . Let A be an algebra over S_2 . Then $\text{id}_{\text{the sorts of } A} \cdot f = \text{id}_{\text{the sorts of } A \upharpoonright_{(f,g)} S_1}$.

- (33) Let S_1 be a non empty many sorted signature, S_2 be a non empty subsignature of S_1 , and A be an algebra over S_1 . Then $\text{id}_{\text{the sorts of } A} \upharpoonright \text{the carrier of } S_2 = \text{id}_{\text{the sorts of } A \upharpoonright S_2}$.
- (34) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2 . Let A, B be algebras over S_2 , h_2 be a many sorted function from A into B , and h_1 be a many sorted function from $A \upharpoonright_{(f,g)} S_1$ into $B \upharpoonright_{(f,g)} S_1$. Suppose $h_1 = h_2 \cdot f$. Let o_1 be an operation symbol of S_1 and o_2 be an operation symbol of S_2 . Suppose $o_2 = g(o_1)$ and $\text{Args}(o_2, A) \neq \emptyset$ and $\text{Args}(o_2, B) \neq \emptyset$. Let x_2 be an element of $\text{Args}(o_2, A)$ and x_1 be an element of $\text{Args}(o_1, A \upharpoonright_{(f,g)} S_1)$. If $x_2 = x_1$, then $h_1 \# x_1 = h_2 \# x_2$.
- (35) Let S, S' be non empty non void many sorted signatures and A_1, A_2 be algebras over S . Suppose the sorts of A_1 are transformable to the sorts of A_2 . Let h be a many sorted function from A_1 into A_2 . Suppose h is a homomorphism of A_1 into A_2 . Let f be a function from the carrier of S' into the carrier of S and g be a function. Suppose f and g form morphism between S' and S . Then there exists a many sorted function h' from $A_1 \upharpoonright_{(f,g)} S'$ into $A_2 \upharpoonright_{(f,g)} S'$ such that $h' = h \cdot f$ and h' is a homomorphism of $A_1 \upharpoonright_{(f,g)} S'$ into $A_2 \upharpoonright_{(f,g)} S'$.
- (36) Let S be a non void feasible many sorted signature, S' be a non void subsignature of S , and A_1, A_2 be algebras over S . Suppose the sorts of A_1 are transformable to the sorts of A_2 . Let h be a many sorted function from A_1 into A_2 . Suppose h is a homomorphism of A_1 into A_2 . Then there exists a many sorted function h' from $A_1 \upharpoonright S'$ into $A_2 \upharpoonright S'$ such that $h' = h \upharpoonright \text{the carrier of } S'$ and h' is a homomorphism of $A_1 \upharpoonright S'$ into $A_2 \upharpoonright S'$.
- (37) Let S, S' be non empty non void many sorted signatures, A be a non-empty algebra over S , f be a function from the carrier of S' into the carrier of S , and g be a function. Suppose f and g form morphism between S' and S . Let B be a non-empty algebra over S' . Suppose $B = A \upharpoonright_{(f,g)} S'$. Let s_1, s_2 be sort symbols of S' and t be a function. Suppose t is an elementary translation in B from s_1 into s_2 . Then t is an elementary translation in A from $f(s_1)$ into $f(s_2)$.
- (38) Let S, S' be non empty non void many sorted signatures, f be a function from the carrier of S' into the carrier of S , and g be a function. Suppose f and g form morphism between S' and S . Let s_1, s_2 be sort symbols of S' . If $\text{TranslRel}(S')$ reduces s_1 to s_2 , then $\text{TranslRel}(S)$ reduces $f(s_1)$ to $f(s_2)$.
- (39) Let S, S' be non void non empty many sorted signatures, A be a non-empty algebra over S , f be a function from the carrier of S' into the carrier of S , and g be a function. Suppose f and g form morphism between S' and S . Let B be a non-empty algebra over S' . Suppose $B = A \upharpoonright_{(f,g)} S'$. Let s_1, s_2 be sort symbols of S' . Suppose $\text{TranslRel}(S')$ reduces s_1 to s_2 . Then every translation in B from s_1 into s_2 is a translation in A from $f(s_1)$ into $f(s_2)$.

4. TRANSLATING HOMOMORPHISMS

The scheme *GenFuncEx* deals with a non empty non void many sorted signature \mathcal{A} , a non-empty algebra \mathcal{B} over \mathcal{A} , a non-empty many sorted set \mathcal{C} indexed by the carrier of \mathcal{A} , and a binary functor \mathcal{F} yielding a set, and states that:

There exists a many sorted function h from $\text{Free}(\mathcal{C})$ into \mathcal{B} such that

- (i) h is a homomorphism of $\text{Free}(\mathcal{C})$ into \mathcal{B} , and
- (ii) for every sort symbol s of \mathcal{A} and for every element x of $\mathcal{C}(s)$ holds $h(s)$ (the root tree of $\langle x, s \rangle = \mathcal{F}(x, s)$)

provided the parameters meet the following requirement:

- For every sort symbol s of \mathcal{A} and for every element x of $\mathcal{C}(s)$ holds $\mathcal{F}(x, s) \in (\text{the sorts of } \mathcal{B})(s)$.

Next we state the proposition

- (40) Let I be a set, A, B be many sorted sets indexed by I , C be a many sorted subset indexed by A , F be a many sorted function from A into B , and i be a set. Suppose $i \in I$. Let f, g be functions. Suppose $f = F(i)$ and $g = (F \upharpoonright C)(i)$. Let x be a set. If $x \in C(i)$, then $g(x) = f(x)$.

Let S be a non void non empty many sorted signature and let X be a non-empty many sorted set indexed by the carrier of S . Observe that $\text{FreeGenerator}(X)$ is non-empty.

Let S_1, S_2 be non empty non void many sorted signatures, let X be a non-empty many sorted set indexed by the carrier of S_2 , let f be a function from the carrier of S_1 into the carrier of S_2 , and let g be a function. Let us assume that f and g form morphism between S_1 and S_2 . The functor $\text{hom}(f, g, X, S_1, S_2)$ yielding a many sorted function from $\text{Free}(X \cdot f)$ into $\text{Free}(X) \downarrow_{(f,g)} S_1$ is defined by the conditions (Def. 5).

- (Def. 5)(i) $\text{hom}(f, g, X, S_1, S_2)$ is a homomorphism of $\text{Free}(X \cdot f)$ into $\text{Free}(X) \downarrow_{(f,g)} S_1$, and
(ii) for every sort symbol s of S_1 and for every element x of $(X \cdot f)(s)$ holds
 $(\text{hom}(f, g, X, S_1, S_2))(s)(\text{the root tree of } \langle x, s \rangle) = \text{the root tree of } \langle x, f(s) \rangle$.

One can prove the following propositions:

- (41) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2 , f be a function from the carrier of S_1 into the carrier of S_2 , and g be a function. Suppose f and g form morphism between S_1 and S_2 . Let o be an operation symbol of S_1 , p be an element of $\text{Args}(o, \text{Free}(X \cdot f))$, and q be a finite sequence. Suppose $q = \text{hom}(f, g, X, S_1, S_2) \# p$. Then $(\text{hom}(f, g, X, S_1, S_2))(\text{the result sort of } o)(\langle o, \text{the carrier of } S_1 \rangle\text{-tree}(p)) = \langle g(o), \text{the carrier of } S_2 \rangle\text{-tree}(q)$.
- (42) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2 , f be a function from the carrier of S_1 into the carrier of S_2 , and g be a function. Suppose f and g form morphism between S_1 and S_2 . Let t be a term of S_1 over $X \cdot f$. Then $(\text{hom}(f, g, X, S_1, S_2))(\text{the sort of } t)(t)$ is a compound term of S_2 over X if and only if t is a compound term of S_1 over $X \cdot f$.
- (43) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2 , f be a function from the carrier of S_1 into the carrier of S_2 , and g be an one-to-one function. Suppose f and g form morphism between S_1 and S_2 . Then $\text{hom}(f, g, X, S_1, S_2)$ is a monomorphism of $\text{Free}(X \cdot f)$ into $\text{Free}(X) \downarrow_{(f,g)} S_1$.
- (44) Let S be a non void non empty many sorted signature and X be a non-empty many sorted set indexed by the carrier of S . Then $\text{hom}(\text{id}_{\text{the carrier of } S}, \text{id}_{\text{the operation symbols of } S}, X, S, S) = \text{id}_{\text{the sorts of } \text{Free}(X)}$.
- (45) Let S_1, S_2, S_3 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_3 , f_1 be a function from the carrier of S_1 into the carrier of S_2 , and g_1 be a function. Suppose f_1 and g_1 form morphism between S_1 and S_2 . Let f_2 be a function from the carrier of S_2 into the carrier of S_3 and g_2 be a function. Suppose f_2 and g_2 form morphism between S_2 and S_3 . Then $\text{hom}(f_2 \cdot f_1, g_2 \cdot g_1, X, S_1, S_3) = (\text{hom}(f_2, g_2, X, S_2, S_3) \circ \text{hom}(f_1, g_1, X \cdot f_2, S_1, S_2))$.

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