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Homotheties and Shears in Affine Planes[1]

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Summary. We study connections between Major Desargues Axiom and the transitivity of group of homotheties. A formal proof of the theorem which establishes an equivalence of these two properties of affine planes is given. We also study connections between trapezium version of Major Desargues Axiom and the existence of the shears in affine planes. The article contains investigations on "Scherungssatz".

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The articles [8], [2], [1], [3], [5], [6], [4], and [7] provide the notation and terminology for this paper.

For simplicity, we use the following convention: A_1 is an affine plane, a, b, o, p, p', q, q', x, y are elements of A_1, M, K are subsets of A_1, and f is a permutation of the carrier of A_1.

One can prove the following four propositions:

(1) Suppose that not $\mathbf{L}(o,a,p)$ and $\mathbf{L}(o,a,b)$ and $\mathbf{L}(o,a,x)$ and $\mathbf{L}(o,a,y)$ and $\mathbf{L}(o,p,p')$ and $\mathbf{L}(o,p,q)$ and $\mathbf{L}(o,p,q')$ and $p \neq q$ and $a \neq x$ and $o \neq q$ and $o \neq x$ and $a,p \upuparrows b,p'$ and $a,q \upuparrows b,q'$ and $x,p \upuparrows y,p'$ and A_1 satisfies **DES**. Then $x,q \upuparrows y,q'$.

(2) Suppose that for all o, a, b such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$ there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$. Then A_1 satisfies **DES**.

(3) Suppose A_1 satisfies **DES**. Let given o, a, b. Suppose $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$. Then there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$.

(4) A_1 satisfies **DES** if and only if for all o, a, b such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$ there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$.

Let us consider A_1, f, K. We say that f is Sc K if and only if:

(Def. 1) f is a collineation and K is a line and for every x such that $x \in K$ holds $f(x) = x$ and for every x holds x, $f(x) \parallel K$.

One can prove the following propositions:

(5) If f is Sc K and $f(p) = p$ and $p \notin K$, then $f = \mathrm{id}_{\text{the carrier of } A_1}$.

(6) If for all a, b, K such that $a, b \parallel K$ and $a \notin K$ there exists f such that f is Sc K and $f(a) = b$, then A_1 satisfies **TDES**.

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(7) Suppose that $K \mathbin{/\!/} M$ and $p \in K$ and $q \in K$ and $p' \in K$ and $q' \in K$ and A_1 satisfies **TDES** and $a \in M$ and $b \in M$ and $x \in M$ and $y \in M$ and $a \neq b$ and $q \neq p$ and $p,a \upuparrows p',x$ and $p,b \upuparrows p',y$ and $q,a \upuparrows q',x$. Then $q,b \upuparrows q',y$.

(8) If a, $b \mathbin{/\!/} K$ and $a \notin K$ and A_1 satisfies **TDES**, then there exists f such that f is Sc K and $f(a) = b$.

(9) A_1 satisfies **TDES** if and only if for all a, b, K such that a, $b \mathbin{/\!/} K$ and $a \notin K$ there exists f such that f is Sc K and $f(a) = b$.

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