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Hessenberg Theorem[1]

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Summary. We prove the Hessenberg theorem which states that every Pappian projective space is Desarguesian.

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The articles [2] and [1] provide the notation and terminology for this paper.

We adopt the following convention: P_1 denotes a projective space defined in terms of collinearity and a, a', a_1, a_2, a_3, b, b', b_1, b_2, c, c_1, c_3, d, d', e, o, p, p_1, p_2, p_3, r, q, q_1, q_2, q_3, x, y denote elements of P_1.

One can prove the following propositions:

(3)[1] Suppose a, b and c are collinear. Then

(i) b, c and a are collinear,

(ii) c, a and b are collinear,

(iii) b, a and c are collinear,

(iv) a, c and b are collinear, and

(v) c, b and a are collinear.

(4) If $a \neq b$ and a, b and c are collinear and a, b and d are collinear, then a, c and d are collinear.

(5) Suppose $p \neq q$ and a, b and p are collinear and a, b and q are collinear and p, q and r are collinear. Then a, b and r are collinear.

(6) If $p \neq q$, then there exists r such that p, q and r are not collinear.

(7) There exist q, r such that p, q and r are not collinear.

(8) If a, b and c are not collinear and a, b and b' are collinear and $a \neq b'$, then a, b' and c are not collinear.

(9) If a, b and c are not collinear and a, b and d are collinear and a, c and d are collinear, then $a = d$.

(10) Suppose that o, a and d are not collinear and o, d and d' are collinear and a, d and x are collinear and $d \neq d'$ and a', d' and x are collinear and o, a and a' are collinear and $o \neq a'$. Then $x \neq d$.

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[1] The propositions (1) and (2) have been removed.

(12)[2] Suppose that a_1, a_2 and a_3 are not collinear and a_1, a_2 and c_3 are collinear and a_2, a_3 and c_1 are collinear and a_1, a_3 and x are collinear and c_1, c_3 and x are collinear and $c_3 \neq a_1$ and $c_3 \neq a_2$ and $c_1 \neq a_2$ and $c_1 \neq a_3$. Then $a_1 \neq x$ and $a_3 \neq x$.

(13) Suppose a, b and c are not collinear and a, b and d are collinear and c, e and d are collinear and $e \neq c$ and $d \neq a$. Then e, a and c are not collinear.

(14) Suppose p_1, p_2 and q_1 are not collinear and p_1, p_2 and q_2 are collinear and q_1, q_2 and q_3 are collinear and $p_1 \neq q_2$ and $q_2 \neq q_3$. Then p_2, p_1 and q_3 are not collinear.

(15) Suppose p_1, p_2 and q_1 are not collinear and p_1, p_2 and p_3 are collinear and q_1, q_2 and p_3 are collinear and $p_3 \neq q_2$ and $p_2 \neq p_3$. Then p_3, p_2 and q_2 are not collinear.

(16) Suppose p_1, p_2 and q_1 are not collinear and p_1, p_2 and p_3 are collinear and q_1, q_2 and p_1 are collinear and $p_1 \neq p_3$ and $p_1 \neq q_2$. Then p_3, p_1 and q_2 are not collinear.

(17) Suppose that $a_1 \neq a_2$ and $b_1 \neq b_2$ and b_1, b_2 and x are collinear and b_1, b_2 and y are collinear and a_1, a_2 and x are collinear and a_1, a_2 and y are collinear and a_1, a_2 and b_1 are not collinear. Then $x = y$.

(19)[3] Suppose o, a_1 and a_2 are not collinear and o, a_1 and b_1 are collinear and o, a_2 and b_2 are collinear and $o \neq b_1$ and $o \neq b_2$. Then o, b_1 and b_2 are not collinear.

We adopt the following convention: P_1 is a Pappian projective plane defined in terms of collinearity and a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3, o, p_1, p_2, p_3, q_1, q_2, q_3, r_1, r_2, r_3 are elements of P_1.

One can prove the following propositions:

(20) Suppose that $p_2 \neq p_3$ and $p_1 \neq p_3$ and $q_2 \neq q_3$ and $q_1 \neq q_2$ and $q_1 \neq q_3$ and p_1, p_2 and q_1 are not collinear and p_1, p_2 and p_3 are collinear and q_1, q_2 and q_3 are collinear and p_1, q_2 and r_3 are collinear and q_1, p_2 and r_3 are collinear and p_1, q_3 and r_2 are collinear and p_3, q_1 and r_2 are collinear and p_2, q_3 and r_1 are collinear and p_3, q_2 and r_1 are collinear. Then r_1, r_2 and r_3 are collinear.

(21) Suppose that $o \neq b_1$ and $a_1 \neq b_1$ and $o \neq b_2$ and $a_2 \neq b_2$ and $o \neq b_3$ and $a_3 \neq b_3$ and o, a_1 and a_2 are not collinear and o, a_1 and a_3 are not collinear and o, a_2 and a_3 are not collinear and a_1, a_2 and c_3 are collinear and b_1, b_2 and c_3 are collinear and a_2, a_3 and c_1 are collinear and b_2, b_3 and c_1 are collinear and a_1, a_3 and c_2 are collinear and b_1, b_3 and c_2 are collinear and o, a_1 and b_1 are collinear and o, a_2 and b_2 are collinear and o, a_3 and b_3 are collinear. Then c_1, c_2 and c_3 are collinear.

One can check that every projective space defined in terms of collinearity which is Pappian is also Desarguesian.

References

[1] Wojciech Leończuk and Krzysztof Prażmowski. Projective spaces. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/anproj_2.html`.

[2] Wojciech Skaba. The collinearity structure. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/collsp.html`.

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[2] The proposition (11) has been removed.
[3] The proposition (18) has been removed.