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Heine–Borel's Covering Theorem

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Summary. Heine–Borel's covering theorem, also known as Borel–Lebesgue theorem ([5]), is proved. Some useful theorems about real inequalities, intervals, sequences and notion of power sequence which are necessary for the theorem are also proved.

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The articles [20], [22], [2], [21], [3], [1], [23], [7], [11], [9], [6], [16], [8], [4], [17], [13], [12], [14], [18], [19], [15], and [10] provide the notation and terminology for this paper.

We adopt the following rules: a, b, x, y, z are real numbers and k, n are natural numbers.

The following propositions are true:

(1) Let A be a subspace of the metric space of real numbers and p, q be points of A. If $x = p$ and $y = q$, then $\rho(p,q) = |x-y|$.

(2) If $x \leq y$ and $y \leq z$, then $[x,y] \cup [y,z] = [x,z]$.

(3) If $x \geq 0$ and $a+x \leq b$, then $a \leq b$.

(4) If $x \geq 0$ and $a-x \geq b$, then $a \geq b$.

(5) If $x > 0$, then $x^k > 0$.

In the sequel s_1 is a sequence of real numbers.

One can prove the following proposition

(6) If s_1 is increasing and $\operatorname{rng} s_1 \subseteq \mathbb{N}$, then $n \leq s_1(n)$.

Let us consider s_1, k. The functor k^{s_1} yields a sequence of real numbers and is defined by:

(Def. 1) For every n holds $k^{s_1}(n) = k^{s_1(n)}$.

One can prove the following propositions:

(7) $2^n \geq n+1$.

(8) $2^n > n$.

(9) If s_1 is divergent to $+\infty$, then 2^{s_1} is divergent to $+\infty$.

(10) For every topological space T such that the carrier of T is finite holds T is compact.

(11) If $a \leq b$, then $[a, b]_{\mathrm{T}}$ is compact.

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