JOURNAL OF FORMALIZED MATHEMATICS
Volume 3, Released 1991, Published 2003
Inst. of Computer Science, Univ. of Białystok

Categories of Groups

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Summary. We define the category of groups and its subcategories: category of Abelian groups and category of groups with the operator of $1/2$. The carriers of the groups are included in a universum. The universum is a parameter of the categories.

MML Identifier: GRCAT_1.

WWW: http://mizar.org/JFM/Vol3/grcat_1.html

The articles [14], [6], [17], [18], [2], [15], [1], [16], [8], [10], [5], [4], [13], [3], [7], [12], [9], and [11] provide the notation and terminology for this paper.

In this paper x, y denote sets, D denotes a non empty set, and U_1 denotes a universal class.

One can prove the following three propositions:

(2)[1] Let X, Y, A be sets and z be a set. Suppose $z \in A$ and $A \subseteq [:X, Y:]$. Then there exists an element x of X and there exists an element y of Y such that $z = \langle x, y \rangle$.

(3) For all elements u_1, u_2, u_3, u_4 of U_1 holds $\langle u_1, u_2, u_3 \rangle$ is an element of U_1 and $\langle u_1, u_2, u_3, u_4 \rangle$ is an element of U_1.

(4) For all x, y such that $x \in y$ and $y \in U_1$ holds $x \in U_1$.

In this article we present several logical schemes. The scheme *PartLambda2* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding a set, and a binary predicate $\mathcal{P}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that for all x, y holds $\langle x, y \rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x,y]$ and for all x, y such that $\langle x, y \rangle \in \operatorname{dom} f$ holds $f(\langle x, y \rangle) = \mathcal{F}(x,y)$

provided the following requirement is met:

- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x,y]$ holds $\mathcal{F}(x,y) \in \mathcal{C}$.

The scheme *PartLambda2D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding a set, and a binary predicate $\mathcal{P}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ holds $\langle x, y \rangle \in \operatorname{dom} f$ iff $\mathcal{P}[x,y]$, and

(ii) for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ such that $\langle x, y \rangle \in \operatorname{dom} f$ holds $f(\langle x, y \rangle) = \mathcal{F}(x,y)$

provided the following condition is satisfied:

- For every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ such that $\mathcal{P}[x,y]$ holds $\mathcal{F}(x,y) \in \mathcal{C}$.

One can prove the following propositions:

[1] The proposition (1) has been removed.

(5) $\mathrm{op}_2(\emptyset, \emptyset) = \emptyset$ and $\mathrm{op}_1(\emptyset) = \emptyset$ and $\mathrm{op}_0 = \emptyset$.

(6) $\{\emptyset\} \in U_1$ and $\langle\{\emptyset\}, \{\emptyset\}\rangle \in U_1$ and $[:\{\emptyset\}, \{\emptyset\}:] \in U_1$ and $\mathrm{op}_2 \in U_1$ and $\mathrm{op}_1 \in U_1$.

(7) $\langle\{\emptyset\}, \mathrm{op}_2, \mathrm{Extract}(\emptyset)\rangle$ is midpoint operator.

Let us note that the trivial loop is midpoint operator.
One can prove the following proposition

(8)(i) For every element x of the trivial loop holds $x = \emptyset$,

(ii) for all elements x, y of the trivial loop holds $x + y = \emptyset$,

(iii) for every element x of the trivial loop holds $-x = \emptyset$, and

(iv) $0_{\text{the trivial loop}} = \emptyset$.

In the sequel C is a category and O is a non empty subset of the objects of C.

Let us consider C, O. The functor $\mathrm{Morphs}\, O$ yields a subset of the morphisms of C and is defined by:

(Def. 5)[2] $\mathrm{Morphs}\, O = \bigcup\{\hom(a, b); a$ ranges over objects of C, b ranges over objects of C: $a \in O \wedge b \in O\}$.

Let us consider C, O. Observe that $\mathrm{Morphs}\, O$ is non empty.

Let us consider C, O. The functor $\mathrm{dom}\, O$ yields a function from $\mathrm{Morphs}\, O$ into O and is defined by:

(Def. 6) $\mathrm{dom}\, O = (\text{the dom-map of } C) \restriction \mathrm{Morphs}\, O$.

The functor $\mathrm{cod}\, O$ yielding a function from $\mathrm{Morphs}\, O$ into O is defined by:

(Def. 7) $\mathrm{cod}\, O = (\text{the cod-map of } C) \restriction \mathrm{Morphs}\, O$.

The functor $\mathrm{comp}\, O$ yielding a partial function from $[:\mathrm{Morphs}\, O, \mathrm{Morphs}\, O:]$ to $\mathrm{Morphs}\, O$ is defined as follows:

(Def. 8) $\mathrm{comp}\, O = (\text{the composition of } C) \restriction ([:\mathrm{Morphs}\, O, \mathrm{Morphs}\, O:]$ **qua** set$)$.

The functor I_O yields a function from O into $\mathrm{Morphs}\, O$ and is defined as follows:

(Def. 9) $\mathrm{I}_O = (\text{the id-map of } C) \restriction O$.

Next we state the proposition

(9) $\langle O, \mathrm{Morphs}\, O, \mathrm{dom}\, O, \mathrm{cod}\, O, \mathrm{comp}\, O, \mathrm{I}_O\rangle$ is full subcategory of C.

Let us consider C, O. The functor $\mathrm{cat}\, O$ yields a subcategory of C and is defined as follows:

(Def. 10) $\mathrm{cat}\, O = \langle O, \mathrm{Morphs}\, O, \mathrm{dom}\, O, \mathrm{cod}\, O, \mathrm{comp}\, O, \mathrm{I}_O\rangle$.

Let us consider C, O. One can verify that $\mathrm{cat}\, O$ is strict.
Next we state the proposition

(10) The objects of $\mathrm{cat}\, O = O$.

Let G be a 1-sorted structure. The functor id_G yielding a map from G into G is defined as follows:

(Def. 11) $\mathrm{id}_G = \mathrm{id}_{\text{the carrier of } G}$.

Next we state two propositions:

(11) For every non empty 1-sorted structure G and for every element x of G holds $\mathrm{id}_G(x) = x$.

[2] The definitions (Def. 1)–(Def. 4) have been removed.

(12) Let G be a 1-sorted structure, H be a non empty 1-sorted structure, and f be a map from G into H. Then $f \cdot \mathrm{id}_G = f$ and $\mathrm{id}_H \cdot f = f$.

Let G, H be non empty zero structures. The functor $\mathrm{ZeroMap}(G,H)$ yields a map from G into H and is defined as follows:

(Def. 12) $\mathrm{ZeroMap}(G,H) = (\text{the carrier of } G) \longmapsto 0_H$.

Let G, H be non empty loop structures and let f be a map from G into H. We say that f is additive if and only if:

(Def. 13) For all elements x, y of G holds $f(x+y) = f(x) + f(y)$.

One can prove the following four propositions:

(13) $\mathrm{comp}(\text{the trivial loop}) = \mathrm{op}_1$.

(14) Let G_1, G_2, G_3 be non empty loop structures, f be a map from G_1 into G_2, and g be a map from G_2 into G_3. If f is additive and g is additive, then $g \cdot f$ is additive.

(15) For every non empty zero structure G and for every non empty loop structure H and for every element x of G holds $(\mathrm{ZeroMap}(G,H))(x) = 0_H$.

(16) For every non empty loop structure G and for every right zeroed non empty loop structure H holds $\mathrm{ZeroMap}(G,H)$ is additive.

In the sequel G, H denote groups.

We consider group morphism structures as systems

⟨ a dom-map, a cod-map, a `Fun` ⟩,

where the dom-map and the cod-map are groups and the `Fun` is a map from the dom-map into the cod-map.

Let f be a group morphism structure. The functor $\mathrm{dom}\, f$ yielding a group is defined by:

(Def. 14) $\mathrm{dom}\, f = \text{the dom-map of } f$.

The functor $\mathrm{cod}\, f$ yields a group and is defined as follows:

(Def. 15) $\mathrm{cod}\, f = \text{the cod-map of } f$.

Let f be a group morphism structure. The functor $\mathrm{fun}\, f$ yields a map from $\mathrm{dom}\, f$ into $\mathrm{cod}\, f$ and is defined by:

(Def. 16) $\mathrm{fun}\, f = \text{the } \mathtt{Fun} \text{ of } f$.

We now state the proposition

(17) Let f be a group morphism structure, G_1, G_2 be groups, and f_0 be a map from G_1 into G_2. If $f = \langle G_1, G_2, f_0 \rangle$, then $\mathrm{dom}\, f = G_1$ and $\mathrm{cod}\, f = G_2$ and $\mathrm{fun}\, f = f_0$.

Let us consider G, H. The functor $\mathrm{ZERO}(G,H)$ yielding a group morphism structure is defined as follows:

(Def. 17) $\mathrm{ZERO}(G,H) = \langle G, H, \mathrm{ZeroMap}(G,H) \rangle$.

Let us consider G, H. One can verify that $\mathrm{ZERO}(G,H)$ is strict.

Let I_1 be a group morphism structure. We say that I_1 is morphism of groups-like if and only if:

(Def. 18) $\mathrm{fun}\, I_1$ is additive.

One can check that there exists a group morphism structure which is strict and morphism of groups-like.

A morphism of groups is a morphism of groups-like group morphism structure.

Next we state the proposition

(18) For every morphism F of groups holds the `Fun` of F is additive.

Let us consider G, H. Observe that $\mathrm{ZERO}(G,H)$ is morphism of groups-like.
Let us consider G, H. A morphism of groups is said to be a morphism from G to H if:

(Def. 19) $\mathrm{dom}\,\mathrm{it} = G$ and $\mathrm{cod}\,\mathrm{it} = H$.

Let us consider G, H. Note that there exists a morphism from G to H which is strict.
One can prove the following three propositions:

(19) Let f be a strict group morphism structure. Suppose $\mathrm{dom}\, f = G$ and $\mathrm{cod}\, f = H$ and $\mathrm{fun}\, f$ is additive. Then f is a strict morphism from G to H.

(20) For every map f from G into H such that f is additive holds $\langle G, H, f\rangle$ is a strict morphism from G to H.

(21) For every non empty loop structure G holds id_G is additive.

Let us consider G. The functor I_G yielding a morphism from G to G is defined by:

(Def. 20) $\mathrm{I}_G = \langle G, G, \mathrm{id}_G\rangle$.

Let us consider G. Note that I_G is strict.
Let us consider G, H. Then $\mathrm{ZERO}(G,H)$ is a strict morphism from G to H.
One can prove the following propositions:

(22) Let F be a morphism from G to H. Then there exists a map f from G into H such that the group morphism structure of $F = \langle G, H, f\rangle$ and f is additive.

(23) For every strict morphism F from G to H there exists a map f from G into H such that $F = \langle G, H, f\rangle$.

(24) For every morphism F of groups there exist G, H such that F is a morphism from G to H.

(25) Let F be a strict morphism of groups. Then there exist groups G, H and there exists a map f from G into H such that F is a morphism from G to H and $F = \langle G, H, f\rangle$ and f is additive.

(26) Let g, f be morphisms of groups. Suppose $\mathrm{dom}\, g = \mathrm{cod}\, f$. Then there exist groups G_1, G_2, G_3 such that g is a morphism from G_2 to G_3 and f is a morphism from G_1 to G_2.

Let G, F be morphisms of groups. Let us assume that $\mathrm{dom}\, G = \mathrm{cod}\, F$. The functor $G \cdot F$ yielding a strict morphism of groups is defined by the condition (Def. 21).

(Def. 21) Let G_1, G_2, G_3 be groups, g be a map from G_2 into G_3, and f be a map from G_1 into G_2. Suppose the group morphism structure of $G = \langle G_2, G_3, g\rangle$ and the group morphism structure of $F = \langle G_1, G_2, f\rangle$. Then $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

Next we state the proposition

(28)[3] Let G_1, G_2, G_3 be groups, G be a morphism from G_2 to G_3, and F be a morphism from G_1 to G_2. Then $G \cdot F$ is a morphism from G_1 to G_3.

Let G_1, G_2, G_3 be groups, let G be a morphism from G_2 to G_3, and let F be a morphism from G_1 to G_2. Then $G \cdot F$ is a strict morphism from G_1 to G_3.
The following propositions are true:

(29) Let G_1, G_2, G_3 be groups, G be a morphism from G_2 to G_3, F be a morphism from G_1 to G_2, g be a map from G_2 into G_3, and f be a map from G_1 into G_2. If $G = \langle G_2, G_3, g\rangle$ and $F = \langle G_1, G_2, f\rangle$, then $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

[3] The proposition (27) has been removed.

(30) Let f, g be strict morphisms of groups. Suppose $\mathrm{dom}\,g = \mathrm{cod}\,f$. Then there exist groups G_1, G_2, G_3 and there exists a map f_0 from G_1 into G_2 and there exists a map g_0 from G_2 into G_3 such that $f = \langle G_1, G_2, f_0 \rangle$ and $g = \langle G_2, G_3, g_0 \rangle$ and $g \cdot f = \langle G_1, G_3, g_0 \cdot f_0 \rangle$.

(31) For all strict morphisms f, g of groups such that $\mathrm{dom}\,g = \mathrm{cod}\,f$ holds $\mathrm{dom}(g \cdot f) = \mathrm{dom}\,f$ and $\mathrm{cod}(g \cdot f) = \mathrm{cod}\,g$.

(32) Let G_1, G_2, G_3, G_4 be groups, f be a strict morphism from G_1 to G_2, g be a strict morphism from G_2 to G_3, and h be a strict morphism from G_3 to G_4. Then $h \cdot (g \cdot f) = (h \cdot g) \cdot f$.

(33) For all strict morphisms f, g, h of groups such that $\mathrm{dom}\,h = \mathrm{cod}\,g$ and $\mathrm{dom}\,g = \mathrm{cod}\,f$ holds $h \cdot (g \cdot f) = (h \cdot g) \cdot f$.

(34)(i) $\mathrm{dom}(\mathrm{I}_G) = G$,

(ii) $\mathrm{cod}(\mathrm{I}_G) = G$,

(iii) for every strict morphism f of groups such that $\mathrm{cod}\,f = G$ holds $\mathrm{I}_G \cdot f = f$, and

(iv) for every strict morphism g of groups such that $\mathrm{dom}\,g = G$ holds $g \cdot \mathrm{I}_G = g$.

Let I_1 be a set. We say that I_1 is non empty set of groups-like if and only if:

(Def. 22) For every set x such that $x \in I_1$ holds x is a strict group.

Let us note that there exists a set which is non empty set of groups-like and non empty.
A non empty set of groups is a non empty set of groups-like non empty set.
In the sequel V is a non empty set of groups.
Let us consider V. We see that the element of V is a group.
Let us consider V. One can verify that there exists an element of V which is strict.
Let I_1 be a set. We say that I_1 is non empty set of morphisms of groups-like if and only if:

(Def. 23) For every set x such that $x \in I_1$ holds x is a strict morphism of groups.

Let us mention that there exists a set which is non empty set of morphisms of groups-like and non empty.

A non empty set of morphisms of groups is a non empty set of morphisms of groups-like non empty set.

Let M be a non empty set of morphisms of groups. We see that the element of M is a morphism of groups.

Let M be a non empty set of morphisms of groups. Note that there exists an element of M which is strict.

Next we state the proposition

(37)[4] For every strict morphism f of groups holds $\{f\}$ is a non empty set of morphisms of groups.

Let us consider G, H. A non empty set of morphisms of groups is said to be a non empty set of morphisms from G into H if:

(Def. 24) Every element of it is a strict morphism from G to H.

The following propositions are true:

(38) D is a non empty set of morphisms from G into H iff every element of D is a strict morphism from G to H.

(39) For every strict morphism f from G to H holds $\{f\}$ is a non empty set of morphisms from G into H.

Let G, H be 1-sorted structures. Set of maps from G into H is defined by:

[4] The propositions (35) and (36) have been removed.

(Def. 25) For every set x such that $x \in$ it holds x is a map from G into H.

Let G, H be 1-sorted structures. The functor $\mathrm{Maps}(G,H)$ yielding a set of maps from G into H is defined by:

(Def. 26) $\mathrm{Maps}(G,H) =$ (the carrier of H)$^{\text{the carrier of } G}$.

Let G be a 1-sorted structure and let H be a non empty 1-sorted structure. Observe that $\mathrm{Maps}(G,H)$ is non empty.

Let G be a 1-sorted structure and let H be a non empty 1-sorted structure. Observe that there exists a set of maps from G into H which is non empty.

Let G be a 1-sorted structure, let H be a non empty 1-sorted structure, and let M be a non empty set of maps from G into H. We see that the element of M is a map from G into H.

Let us consider G, H. The functor $\mathrm{Morphs}(G,H)$ yielding a non empty set of morphisms from G into H is defined as follows:

(Def. 27) $x \in \mathrm{Morphs}(G,H)$ iff x is a strict morphism from G to H.

Let us consider G, H and let M be a non empty set of morphisms from G into H. We see that the element of M is a morphism from G to H.

Let us consider G, H and let M be a non empty set of morphisms from G into H. Note that there exists an element of M which is strict.

Let us consider x, y. The predicate $\mathrm{P_{ob}}\, x,y$ is defined by the condition (Def. 28).

(Def. 28) There exist sets x_1, x_2, x_3, x_4 such that

(i) $x = \langle x_1, x_2, x_3, x_4 \rangle$, and

(ii) there exists a strict group G such that $y = G$ and $x_1 =$ the carrier of G and $x_2 =$ the addition of G and $x_3 = \mathrm{comp}\, G$ and $x_4 =$ the zero of G.

One can prove the following two propositions:

(40) For all sets x, y_1, y_2 such that $\mathrm{P_{ob}}\, x,y_1$ and $\mathrm{P_{ob}}\, x,y_2$ holds $y_1 = y_2$.

(41) There exists x such that $x \in U_1$ and $\mathrm{P_{ob}}\, x,$ the trivial loop.

Let us consider U_1. The functor $\mathrm{GroupObj}(U_1)$ yielding a set is defined as follows:

(Def. 29) For every y holds $y \in \mathrm{GroupObj}(U_1)$ iff there exists x such that $x \in U_1$ and $\mathrm{P_{ob}}\, x,y$.

We now state the proposition

(42) The trivial loop $\in \mathrm{GroupObj}(U_1)$.

Let us consider U_1. One can check that $\mathrm{GroupObj}(U_1)$ is non empty.

One can prove the following proposition

(43) Every element of $\mathrm{GroupObj}(U_1)$ is a strict group.

Let us consider U_1. Note that $\mathrm{GroupObj}(U_1)$ is non empty set of groups-like.

Let us consider V. The functor $\mathrm{Morphs}\, V$ yielding a non empty set of morphisms of groups is defined as follows:

(Def. 30) For every x holds $x \in \mathrm{Morphs}\, V$ iff there exist strict elements G, H of V such that x is a strict morphism from G to H.

Let us consider V and let F be an element of $\mathrm{Morphs}\, V$. Then $\mathrm{dom}\, F$ is a strict element of V. Then $\mathrm{cod}\, F$ is a strict element of V.

Let us consider V and let G be an element of V. The functor I_G yielding a strict element of $\mathrm{Morphs}\, V$ is defined by:

(Def. 31) $\mathrm{I}_G = \mathrm{I}_G$.

Let us consider V. The functor $\mathrm{dom}\,V$ yields a function from Morphs V into V and is defined by:

(Def. 32) For every element f of Morphs V holds $(\mathrm{dom}\,V)(f) = \mathrm{dom}\,f$.

The functor $\mathrm{cod}\,V$ yielding a function from Morphs V into V is defined by:

(Def. 33) For every element f of Morphs V holds $(\mathrm{cod}\,V)(f) = \mathrm{cod}\,f$.

The functor I_V yields a function from V into Morphs V and is defined as follows:

(Def. 34) For every element G of V holds $\mathrm{I}_V(G) = \mathrm{I}_G$.

The following two propositions are true:

(44) Let g, f be elements of Morphs V. Suppose $\mathrm{dom}\,g = \mathrm{cod}\,f$. Then there exist strict elements G_1, G_2, G_3 of V such that g is a morphism from G_2 to G_3 and f is a morphism from G_1 to G_2.

(45) For all elements g, f of Morphs V such that $\mathrm{dom}\,g = \mathrm{cod}\,f$ holds $g \cdot f \in$ Morphs V.

Let us consider V. The functor $\mathrm{comp}\,V$ yields a partial function from $[:\mathrm{Morphs}\,V, \mathrm{Morphs}\,V:]$ to Morphs V and is defined by the conditions (Def. 35).

(Def. 35)(i) For all elements g, f of Morphs V holds $\langle g, f\rangle \in \mathrm{dom}\,\mathrm{comp}\,V$ iff $\mathrm{dom}\,g = \mathrm{cod}\,f$, and

(ii) for all elements g, f of Morphs V such that $\langle g, f\rangle \in \mathrm{dom}\,\mathrm{comp}\,V$ holds $(\mathrm{comp}\,V)(\langle g, f\rangle) = g \cdot f$.

Let us consider U_1. The functor GroupCat(U_1) yields a category structure and is defined as follows:

(Def. 36) $\mathrm{GroupCat}(U_1) = \langle \mathrm{GroupObj}(U_1), \mathrm{Morphs}\,\mathrm{GroupObj}(U_1), \mathrm{dom}\,\mathrm{GroupObj}(U_1), \mathrm{cod}\,\mathrm{GroupObj}(U_1), \mathrm{comp}\,\mathrm{GroupObj}(U_1), \mathrm{I}_{\mathrm{GroupObj}(U_1)}\rangle$.

Let us consider U_1. Observe that GroupCat(U_1) is strict.

The following propositions are true:

(46) For all morphisms f, g of GroupCat(U_1) holds $\langle g, f\rangle \in$ dom(the composition of GroupCat(U_1)) iff $\mathrm{dom}\,g = \mathrm{cod}\,f$.

(47) Let f be a morphism of GroupCat(U_1), f' be an element of Morphs GroupObj(U_1), b be an object of GroupCat(U_1), and b' be an element of GroupObj(U_1). Then

(i) f is a strict element of Morphs GroupObj(U_1),

(ii) f' is a morphism of GroupCat(U_1),

(iii) b is a strict element of GroupObj(U_1), and

(iv) b' is an object of GroupCat(U_1).

(48) For every object b of GroupCat(U_1) and for every element b' of GroupObj(U_1) such that $b = b'$ holds $\mathrm{id}_b = \mathrm{I}_{b'}$.

(49) For every morphism f of GroupCat(U_1) and for every element f' of Morphs GroupObj(U_1) such that $f = f'$ holds $\mathrm{dom}\,f = \mathrm{dom}\,f'$ and $\mathrm{cod}\,f = \mathrm{cod}\,f'$.

(50) Let f, g be morphisms of GroupCat(U_1) and f', g' be elements of Morphs GroupObj(U_1) such that $f = f'$ and $g = g'$. Then

(i) $\mathrm{dom}\,g = \mathrm{cod}\,f$ iff $\mathrm{dom}\,g' = \mathrm{cod}\,f'$,

(ii) $\mathrm{dom}\,g = \mathrm{cod}\,f$ iff $\langle g', f'\rangle \in \mathrm{dom}\,\mathrm{comp}\,\mathrm{GroupObj}(U_1)$,

(iii) if $\mathrm{dom}\,g = \mathrm{cod}\,f$, then $g \cdot f = g' \cdot f'$,

(iv) $\mathrm{dom}\,f = \mathrm{dom}\,g$ iff $\mathrm{dom}\,f' = \mathrm{dom}\,g'$, and

(v) $\mathrm{cod}\,f = \mathrm{cod}\,g$ iff $\mathrm{cod}\,f' = \mathrm{cod}\,g'$.

Let us consider U_1. One can check that GroupCat(U_1) is category-like.

Let us consider U_1. The functor AbGroupObj(U_1) yields a subset of the objects of GroupCat(U_1) and is defined by:

(Def. 37) AbGroupObj$(U_1) = \{G; G$ ranges over elements of the objects of GroupCat(U_1): $\bigvee_{H:\text{Abelian group}} G = H\}$.

One can prove the following proposition

(51) The trivial loop $\in$ AbGroupObj(U_1).

Let us consider U_1. One can check that AbGroupObj(U_1) is non empty.

Let us consider U_1. The functor AbGroupCat(U_1) yielding a subcategory of GroupCat(U_1) is defined as follows:

(Def. 38) AbGroupCat$(U_1) =$ cat AbGroupObj(U_1).

Let us consider U_1. Observe that AbGroupCat(U_1) is strict.

One can prove the following proposition

(52) The objects of AbGroupCat$(U_1) =$ AbGroupObj(U_1).

Let us consider U_1. The functor $\frac{1}{2}$ GroupObj(U_1) yielding a subset of the objects of AbGroupCat(U_1) is defined by:

(Def. 39) $\frac{1}{2}$ GroupObj$(U_1) = \{G; G$ ranges over elements of the objects of AbGroupCat(U_1): $\bigvee_{H:\text{ midpoint operator Abelian group}} G = H\}$.

Let us consider U_1. Observe that $\frac{1}{2}$ GroupObj(U_1) is non empty.

Let us consider U_1. The functor $\frac{1}{2}$ GroupCat(U_1) yielding a subcategory of AbGroupCat(U_1) is defined by:

(Def. 40) $\frac{1}{2}$ GroupCat$(U_1) =$ cat $\frac{1}{2}$ GroupObj(U_1).

Let us consider U_1. Note that $\frac{1}{2}$ GroupCat(U_1) is strict.

One can prove the following propositions:

(53) The objects of $\frac{1}{2}$ GroupCat$(U_1) = \frac{1}{2}$ GroupObj(U_1).

(54) The trivial loop $\in \frac{1}{2}$ GroupObj(U_1).

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Received October 3, 1991

Published January 2, 2004