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# The First Part of Jordan's Theorem for Special Polygons

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**Summary.** We prove here the first part of Jordan's theorem for special polygons, i.e., the complement of a special polygon is the union of two components (a left component and a right component). At this stage, we do not know if the two components are different from each other.

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The articles [15], [4], [19], [6], [8], [17], [1], [14], [3], [2], [18], [7], [20], [13], [5], [9], [10], [16], [12], and [11] provide the notation and terminology for this paper.

We adopt the following rules: $i$, $j$, $k_1$, $k_2$, $i_1$, $i_2$, $j_1$, $j_2$ are natural numbers and $f$ is a non constant standard special circular sequence.

We now state a number of propositions:

(2)[1] For all $i$, $j$ such that $i \leq \mathrm{len}$ the Go-board of $f$ and $j \leq \mathrm{width}$ the Go-board of $f$ holds Int cell(the Go-board of $f$, $i, j) \subseteq (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(3) Let given $i$, $j$. Suppose $i \leq \mathrm{len}$ the Go-board of $f$ and $j \leq \mathrm{width}$ the Go-board of $f$. Then Down(Int cell(the Go-board of $f$, $i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ = cell(the Go-board of $f$, $i, j) \cap (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(4) Let given $i$, $j$. Suppose $i \leq \mathrm{len}$ the Go-board of $f$ and $j \leq \mathrm{width}$ the Go-board of $f$. Then Down(Int cell(the Go-board of $f$, $i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ is connected and Down(Int cell(the Go-board of $f$, $i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ = Int cell(the Go-board of $f$, $i, j)$.

(5) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = \bigcup\{$Down(Int cell(the Go-board of $f$, $i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) : i \leq \mathrm{len}$ the Go-board of $f \wedge j \leq \mathrm{width}$ the Go-board of $f\}$.

(6) Down(LeftComp$(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) \cup$ Down(RightComp$(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ is a union of components of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and Down(LeftComp$(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ = LeftComp$(f)$ and Down(RightComp$(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ = RightComp$(f)$.

(7) Let given $i_1$, $j_1$, $i_2$, $j_2$. Suppose that

(i) $i_1 \leq \mathrm{len}$ the Go-board of $f$,

(ii) $j_1 \leq \mathrm{width}$ the Go-board of $f$,

(iii) $i_2 \leq \mathrm{len}$ the Go-board of $f$,

[1] The proposition (1) has been removed.

(iv) $j_2 \leq \mathrm{width}\,\text{the Go-board of } f$, and

(v) $i_1$, $j_1$, $i_2$, and $j_2$ are adjacent.

Then $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i_1, j_1) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$ if and only if $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i_2, j_2) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(8) Let $F_1$, $F_2$ be finite sequences of elements of $\mathbb{N}$. Suppose that

(i) $\mathrm{len}\,F_1 = \mathrm{len}\,F_2$,

(ii) there exists $i$ such that $i \in \mathrm{dom}\,F_1$ and $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, (F_1)_i, (F_2)_i) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$,

(iii) for every $i$ such that $1 \leq i$ and $i < \mathrm{len}\,F_1$ holds $(F_1)_i$, $(F_2)_i$, $(F_1)_{i+1}$, and $(F_2)_{i+1}$ are adjacent, and

(iv) for all $i$, $k_1$, $k_2$ such that $i \in \mathrm{dom}\,F_1$ and $k_1 = F_1(i)$ and $k_2 = F_2(i)$ holds $k_1 \leq \mathrm{len}\,\text{the Go-board of } f$ and $k_2 \leq \mathrm{width}\,\text{the Go-board of } f$.

Let given $i$. If $i \in \mathrm{dom}\,F_1$, then $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, (F_1)_i, (F_2)_i) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(9) There exist $i$, $j$ such that $i \leq \mathrm{len}\,\text{the Go-board of } f$ and $j \leq \mathrm{width}\,\text{the Go-board of } f$ and $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(10) For all $i$, $j$ such that $i \leq \mathrm{len}\,\text{the Go-board of } f$ and $j \leq \mathrm{width}\,\text{the Go-board of } f$ holds $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(11) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

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