JOURNAL OF FORMALIZED MATHEMATICS
Volume 8, Released 1996, Published 2003
Inst. of Computer Science, Univ. of Białystok

Adjacency Concept for Pairs of Natural Numbers

Yatsuka Nakamura
Shinshu University
Nagano

Andrzej Trybulec
Warsaw University
Białystok

Summary. First, we introduce the concept of adjacency for a pair of natural numbers. Second, we extend the concept for two pairs of natural numbers. The pairs represent points of a lattice in a plane. We show that if some property is infectious among adjacent points, and some points have the property, then all points have the property.

MML Identifier: GOBRD10.

WWW: http://mizar.org/JFM/Vol8/gobrd10.html

The articles [9], [4], [12], [10], [1], [7], [13], [3], [2], [5], [11], [6], and [8] provide the notation and terminology for this paper.

In this paper i, j, k, k_1, k_2, n, m, i_1, i_2, j_1, j_2 denote natural numbers.

Let us consider i_1, i_2. We say that i_1 and i_2 are adjacent if and only if:

(Def. 1) $i_2 = i_1 + 1$ or $i_1 = i_2 + 1$.

Let us notice that the predicate i_1 and i_2 are adjacent is irreflexive and symmetric.

We now state two propositions:

(1) For all i_1, i_2 such that i_1 and i_2 are adjacent holds $i_1 + 1$ and $i_2 + 1$ are adjacent.

(2) For all i_1, i_2 such that i_1 and i_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ holds $i_1 -' 1$ and $i_2 -' 1$ are adjacent.

Let us consider i_1, j_1, i_2, j_2. We say that i_1, j_1, i_2, and j_2 are adjacent if and only if:

(Def. 2) i_1 and i_2 are adjacent and $j_1 = j_2$ or $i_1 = i_2$ and j_1 and j_2 are adjacent.

One can prove the following two propositions:

(3) For all i_1, i_2, j_1, j_2 such that i_1, j_1, i_2, and j_2 are adjacent holds $i_1 + 1$, $j_1 + 1$, $i_2 + 1$, and $j_2 + 1$ are adjacent.

(4) Let given i_1, i_2, j_1, j_2. Suppose i_1, j_1, i_2, and j_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ and $1 \leq j_1$ and $1 \leq j_2$. Then $i_1 -' 1$, $j_1 -' 1$, $i_2 -' 1$, and $j_2 -' 1$ are adjacent.

Let us consider n, i. Then $n \mapsto i$ is a finite sequence of elements of $\mathbb{N}$ and it can be characterized by the condition:

(Def. 3) $\mathrm{len}(n \mapsto i) = n$ and for every j such that $1 \leq j$ and $j \leq n$ holds $(n \mapsto i)(j) = i$.

The following propositions are true:

(6)[1] Let given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that

(i) $f_1(1) = i$,

(ii) $f_1(\mathrm{len}\, f_1) = j$,

(iii) $\mathrm{len}\, f_1 = (i -' j) + (j -' i) + 1$,

(iv) for all k, k_1 such that $1 \leq k$ and $k \leq \mathrm{len}\, f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and

(v) for every i_1 such that $1 \leq i_1$ and $i_1 < \mathrm{len}\, f_1$ holds $f_1(i_1 + 1) = (f_1)_{i_1} + 1$ or $f_1(i_1) = (f_1)_{i_1+1} + 1$.

(7) Let given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that

(i) $f_1(1) = i$,

(ii) $f_1(\mathrm{len}\, f_1) = j$,

(iii) $\mathrm{len}\, f_1 = (i -' j) + (j -' i) + 1$,

(iv) for all k, k_1 such that $1 \leq k$ and $k \leq \mathrm{len}\, f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and

(v) for every i_1 such that $1 \leq i_1$ and $i_1 < \mathrm{len}\, f_1$ holds $(f_1)_{i_1}$ and $(f_1)_{i_1+1}$ are adjacent.

(8) Let given n, m, i_1, j_1, i_2, j_2. Suppose $i_1 \leq n$ and $j_1 \leq m$ and $i_2 \leq n$ and $j_2 \leq m$. Then there exist finite sequences f_1, f_2 of elements of $\mathbb{N}$ such that

for all i, k_1, k_2 such that $i \in \mathrm{dom}\, f_1$ and $k_1 = f_1(i)$ and $k_2 = f_2(i)$ holds $k_1 \leq n$ and $k_2 \leq m$ and $f_1(1) = i_1$ and $f_1(\mathrm{len}\, f_1) = i_2$ and $f_2(1) = j_1$ and $f_2(\mathrm{len}\, f_2) = j_2$ and $\mathrm{len}\, f_1 = \mathrm{len}\, f_2$ and $\mathrm{len}\, f_1 = (i_1 -' i_2) + (i_2 -' i_1) + (j_1 -' j_2) + (j_2 -' j_1) + 1$ and for every i such that $1 \leq i$ and $i < \mathrm{len}\, f_1$ holds $(f_1)_i$, $(f_2)_i$, $(f_1)_{i+1}$, and $(f_2)_{i+1}$ are adjacent.

In the sequel S denotes a set.
The following proposition is true

(9) Let Y be a subset of S and F be a matrix over 2^S of dimension $n \times m$. Suppose that

(i) there exist i, j such that $i \in \mathrm{Seg}\, n$ and $j \in \mathrm{Seg}\, m$ and $F \circ (i, j) \subseteq Y$, and

(ii) for all i_1, j_1, i_2, j_2 such that $i_1 \in \mathrm{Seg}\, n$ and $i_2 \in \mathrm{Seg}\, n$ and $j_1 \in \mathrm{Seg}\, m$ and $j_2 \in \mathrm{Seg}\, m$ and i_1, j_1, i_2, and j_2 are adjacent holds $F \circ (i_1, j_1) \subseteq Y$ iff $F \circ (i_2, j_2) \subseteq Y$.

Let given i, j. If $i \in \mathrm{Seg}\, n$ and $j \in \mathrm{Seg}\, m$, then $F \circ (i, j) \subseteq Y$.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[3] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[4] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[5] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseq_2.html.

[6] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/matrix_1.html.

[7] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/binarith.html.

[8] Andrzej Trybulec. Binary operations applied to functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funcop_1.html.

[1] The proposition (5) has been removed.

[9] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[10] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

[11] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finseq_4.html`.

[12] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[13] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

Received June 10, 1996

Published January 2, 2004