JOURNAL OF FORMALIZED MATHEMATICS
Volume 7, Released 1995, Published 2003
Inst. of Computer Science, Univ. of Białystok

On the Go-Board of a Standard Special Circular Sequence

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MML Identifier: GOBOARD7.

WWW: http://mizar.org/JFM/Vol7/goboard7.html

The articles [18], [5], [22], [9], [2], [17], [15], [1], [4], [3], [6], [21], [10], [16], [23], [7], [8], [11], [12], [13], [19], [14], and [20] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we adopt the following rules: f denotes a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$, i, j, k, n, i_1, i_2, j_1, j_2 denote natural numbers, r, s, r_1, r_2 denote real numbers, p, q, p_1 denote points of $\mathcal{E}^2_{\rm T}$, and G denotes a Go-board.

We now state a number of propositions:

(1) If $|r_1 - r_2| > s$, then $r_1 + s < r_2$ or $r_2 + s < r_1$.

(2) $|r - s| = 0$ iff $r = s$.

(3) For all points p, p_1, q of $\mathcal{E}^n_{\rm T}$ such that $p + p_1 = q + p_1$ holds $p = q$.

(4) For all points p, p_1, q of $\mathcal{E}^n_{\rm T}$ such that $p_1 + p = p_1 + q$ holds $p = q$.

(5) If $p_1 \in \mathcal{L}(p,q)$ and $p_{\mathbf{1}} = q_{\mathbf{1}}$, then $(p_1)_{\mathbf{1}} = q_{\mathbf{1}}$.

(6) If $p_1 \in \mathcal{L}(p,q)$ and $p_{\mathbf{2}} = q_{\mathbf{2}}$, then $(p_1)_{\mathbf{2}} = q_{\mathbf{2}}$.

(7) $\frac{1}{2} \cdot (p+q) \in \mathcal{L}(p,q)$.

(8) If $p_{\mathbf{1}} = q_{\mathbf{1}}$ and $q_{\mathbf{1}} = (p_1)_{\mathbf{1}}$ and $p_{\mathbf{2}} \leq q_{\mathbf{2}}$ and $q_{\mathbf{2}} \leq (p_1)_{\mathbf{2}}$, then $q \in \mathcal{L}(p, p_1)$.

(9) If $p_{\mathbf{1}} \leq q_{\mathbf{1}}$ and $q_{\mathbf{1}} \leq (p_1)_{\mathbf{1}}$ and $p_{\mathbf{2}} = q_{\mathbf{2}}$ and $q_{\mathbf{2}} = (p_1)_{\mathbf{2}}$, then $q \in \mathcal{L}(p, p_1)$.

(10) Let D be a non empty set, given i, j, and M be a matrix over D. If $1 \leq i$ and $i \leq {\rm len}\, M$ and $1 \leq j$ and $j \leq {\rm width}\, M$, then $\langle i,\ j\rangle \in$ the indices of M.

(11) If $1 \leq i$ and $i+1 \leq {\rm len}\, G$ and $1 \leq j$ and $j+1 \leq {\rm width}\, G$, then $\frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i+1, j+1))) = \frac{1}{2} \cdot ((G \circ (i, j+1)) + (G \circ (i+1, j)))$.

(12) Suppose $\mathcal{L}(f,k)$ is horizontal. Then there exists j such that $1 \leq j$ and $j \leq {\rm width}$ the Go-board of f and for every p such that $p \in \mathcal{L}(f,k)$ holds $p_{\mathbf{2}} =$ (the Go-board of $f \circ (1, j))_{\mathbf{2}}$.

(13) Suppose $\mathcal{L}(f,k)$ is vertical. Then there exists i such that $1 \leq i$ and $i \leq \operatorname{len}$ the Go-board of f and for every p such that $p \in \mathcal{L}(f,k)$ holds $p_{\mathbf{1}} =$ (the Go-board of $f \circ (i,1))_{\mathbf{1}}$.

(14) Suppose f is special and $i \leq \operatorname{len}$ the Go-board of f and $j \leq \operatorname{width}$ the Go-board of f. Then Int cell(the Go-board of f, i, j) misses $\widetilde{\mathcal{L}}(f)$.

2. Segments on a Go-Board

The following propositions are true:

(15) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j+2 \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j), G \circ (i,j+1)) \cap \mathcal{L}(G \circ (i,j+1), G \circ (i,j+2)) = \{G \circ (i,j+1)\}$.

(16) If $1 \leq i$ and $i+2 \leq \operatorname{len} G$ and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j), G \circ (i+1,j)) \cap \mathcal{L}(G \circ (i+1,j), G \circ (i+2,j)) = \{G \circ (i+1,j)\}$.

(17) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j), G \circ (i,j+1)) \cap \mathcal{L}(G \circ (i,j+1), G \circ (i+1,j+1)) = \{G \circ (i,j+1)\}$.

(18) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j+1), G \circ (i+1,j+1)) \cap \mathcal{L}(G \circ (i+1,j), G \circ (i+1,j+1)) = \{G \circ (i+1,j+1)\}$.

(19) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j), G \circ (i+1,j)) \cap \mathcal{L}(G \circ (i,j), G \circ (i,j+1)) = \{G \circ (i,j)\}$.

(20) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G \circ (i,j), G \circ (i+1,j)) \cap \mathcal{L}(G \circ (i+1,j), G \circ (i+1,j+1)) = \{G \circ (i+1,j)\}$.

(21) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1))$ meets $\mathcal{L}(G \circ (i_2,j_2), G \circ (i_2,j_2+1))$. Then $i_1 = i_2$ and $|j_1 - j_2| \leq 1$.

(22) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1+1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1+1,j_1))$ meets $\mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2))$. Then $j_1 = j_2$ and $|i_1 - i_2| \leq 1$.

(23) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1))$ meets $\mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2))$. Then $i_1 = i_2$ or $i_1 = i_2+1$ but $j_1 = j_2$ or $j_1+1 = j_2$.

(24) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1))$ meets $\mathcal{L}(G \circ (i_2,j_2), G \circ (i_2,j_2+1))$. Then

(i) $j_1 = j_2$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1)) = \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2,j_2+1))$, or

(ii) $j_1 = j_2+1$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1)) \cap \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2,j_2+1)) = \{G \circ (i_1,j_1)\}$, or

(iii) $j_1+1 = j_2$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1,j_1+1)) \cap \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2,j_2+1)) = \{G \circ (i_2,j_2)\}$.

(25) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1+1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1+1,j_1))$ meets $\mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2))$. Then

(i) $i_1 = i_2$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1+1,j_1)) = \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2))$, or

(ii) $i_1 = i_2+1$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1+1,j_1)) \cap \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2)) = \{G \circ (i_1,j_1)\}$, or

(iii) $i_1+1 = i_2$ and $\mathcal{L}(G \circ (i_1,j_1), G \circ (i_1+1,j_1)) \cap \mathcal{L}(G \circ (i_2,j_2), G \circ (i_2+1,j_2)) = \{G \circ (i_2,j_2)\}$.

(26) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G \circ (i_1, j_1), G \circ (i_1, j_1+1))$ meets $\mathcal{L}(G \circ (i_2, j_2), G \circ (i_2+1, j_2))$. Then $j_1 = j_2$ and $\mathcal{L}(G \circ (i_1, j_1), G \circ (i_1, j_1+1)) \cap \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2+1, j_2)) = \{G \circ (i_1, j_1)\}$ or $j_1+1 = j_2$ and $\mathcal{L}(G \circ (i_1, j_1), G \circ (i_1, j_1+1)) \cap \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2+1, j_2)) = \{G \circ (i_1, j_1+1)\}$.

(27) Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot ((G \circ (i_1, j_1)) + (G \circ (i_1, j_1+1))) \in \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2, j_2+1))$. Then $i_1 = i_2$ and $j_1 = j_2$.

(28) Suppose $1 \leq i_1$ and $i_1+1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot ((G \circ (i_1, j_1)) + (G \circ (i_1+1, j_1))) \in \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2+1, j_2))$. Then $i_1 = i_2$ and $j_1 = j_2$.

(29) Suppose $1 \leq i_1$ and $i_1+1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$. Then it is not true that there exist i_2, j_2 such that $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot ((G \circ (i_1, j_1)) + (G \circ (i_1+1, j_1))) \in \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2, j_2+1))$.

(30) Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$. Then it is not true that there exist i_2, j_2 such that $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot ((G \circ (i_1, j_1)) + (G \circ (i_1, j_1+1))) \in \mathcal{L}(G \circ (i_2, j_2), G \circ (i_2+1, j_2))$.

3. Standard Special Circular Sequences

In the sequel f denotes a non constant standard special circular sequence.

The following propositions are true:

(31) For every standard non empty finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $i \in \operatorname{dom} f$ and $i+1 \in \operatorname{dom} f$ holds $f_i \neq f_{i+1}$.

(32) There exists i such that $i \in \operatorname{dom} f$ and $(f_i)_{\mathbf{1}} \neq (f_1)_{\mathbf{1}}$.

(33) There exists i such that $i \in \operatorname{dom} f$ and $(f_i)_{\mathbf{2}} \neq (f_1)_{\mathbf{2}}$.

(34) len the Go-board of $f > 1$.

(35) width the Go-board of $f > 1$.

(36) $\operatorname{len} f > 4$.

(37) Let f be a circular s.c.c. finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $\operatorname{len} f > 4$. Let i, j be natural numbers. If $1 \leq i$ and $i < j$ and $j < \operatorname{len} f$, then $f_i \neq f_j$.

(38) For all natural numbers i, j such that $1 \leq i$ and $i < j$ and $j < \operatorname{len} f$ holds $f_i \neq f_j$.

(39) For all natural numbers i, j such that $1 < i$ and $i < j$ and $j \leq \operatorname{len} f$ holds $f_i \neq f_j$.

(40) For every natural number i such that $1 < i$ and $i \leq \operatorname{len} f$ and $f_i = f_1$ holds $i = \operatorname{len} f$.

(41) Suppose that

(i) $1 \leq i$,

(ii) $i \leq$ len the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq$ width the Go-board of f, and

(v) $\frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j)) + (\text{the Go-board of } f \circ (i, j+1))) \in \widetilde{\mathcal{L}}(f)$.

Then there exists k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i, j+1)) = \mathcal{L}(f, k)$.

(42) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j \leq \operatorname{width}$ the Go-board of f, and

(v) $\frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j)) + (\text{the Go-board of } f \circ (i+1, j))) \in \widetilde{\mathcal{L}}(f)$.

Then there exists k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i+1, j)) = \mathcal{L}(f, k)$.

(43) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$.

(44) Suppose that $1 \leq i$ and $i \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 < \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i, j+2)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j+2)$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(45) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(46) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$.

(47) Suppose that $1 \leq i$ and $i+1 < \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+2, j)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i+1, j)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i+2, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(48) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i+1, j)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(49) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$.

(50) Suppose that $1 \leq i$ and $i \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 < \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i, j+2)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+2)$.

(51) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}($the Go-board of $f \circ (i, j),$the Go-board of $f \circ (i, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}($the Go-board of $f \circ (i, j+1),$the Go-board of $f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+1)$.

(52) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}($the Go-board of $f \circ (i, j+1),$the Go-board of $f \circ (i+1, j+1)) = \mathcal{L}(f, k)$ and $\mathcal{L}($the Go-board of $f \circ (i+1, j),$the Go-board of $f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$.

(53) Suppose that $1 \leq i$ and $i+1 < \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}($the Go-board of $f \circ (i, j),$the Go-board of $f \circ (i+1, j)) = \mathcal{L}(f, k)$ and $\mathcal{L}($the Go-board of $f \circ (i+1, j),$the Go-board of $f \circ (i+2, j)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+2, j)$.

(54) Suppose that $1 \leq i$ and $i+1 \leq \operatorname{len}$ the Go-board of f and $1 \leq j$ and $j+1 \leq \operatorname{width}$ the Go-board of f and $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\mathcal{L}($the Go-board of $f \circ (i, j),$the Go-board of $f \circ (i+1, j)) = \mathcal{L}(f, k)$ and $\mathcal{L}($the Go-board of $f \circ (i+1, j),$the Go-board of $f \circ (i+1, j+1)) = \mathcal{L}(f, k+1)$. Then $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+1)$.

(55) Suppose that

(i) $1 \leq i$,

(ii) $i \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 < \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}($the Go-board of $f \circ (i, j),$the Go-board of $f \circ (i, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}($the Go-board of $f \circ (i, j+1),$the Go-board of $f \circ (i, j+2)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 =$ the Go-board of $f \circ (i, j+1)$ but $f_2 =$ the Go-board of $f \circ (i, j)$ and $f_{\operatorname{len} f -' 1} =$ the Go-board of $f \circ (i, j+2)$ or $f_2 =$ the Go-board of $f \circ (i, j+2)$ and $f_{\operatorname{len} f -' 1} =$ the Go-board of $f \circ (i, j)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \operatorname{len} f$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+2)$ or $f_k =$ the Go-board of $f \circ (i, j+2)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(56) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}($the Go-board of $f \circ (i, j),$the Go-board of $f \circ (i, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}($the Go-board of $f \circ (i, j+1),$the Go-board of $f \circ (i+1, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 =$ the Go-board of $f \circ (i, j+1)$ but $f_2 =$ the Go-board of $f \circ (i, j)$ and $f_{\operatorname{len} f -' 1} =$ the Go-board of $f \circ (i+1, j+1)$ or $f_2 =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{\operatorname{len} f -' 1} =$ the Go-board of $f \circ (i, j)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \operatorname{len} f$ and $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$ and $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+1)$ or $f_k =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(57) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \text{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq \text{width}$ the Go-board of f,

(v) $\mathcal{L}$(the Go-board of $f \circ (i, j+1)$, the Go-board of $f \circ (i+1, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}$(the Go-board of $f \circ (i+1, j+1)$, the Go-board of $f \circ (i+1, j)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 =$ the Go-board of $f \circ (i+1, j+1)$ but $f_2 =$ the Go-board of $f \circ (i, j+1)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i+1, j)$ or $f_2 =$ the Go-board of $f \circ (i+1, j)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i, j+1)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$ and $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$ or $f_k =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$.

(58) Suppose that

(i) $1 \leq i$,

(ii) $i+1 < \text{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j \leq \text{width}$ the Go-board of f,

(v) $\mathcal{L}$(the Go-board of $f \circ (i, j)$, the Go-board of $f \circ (i+1, j)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}$(the Go-board of $f \circ (i+1, j)$, the Go-board of $f \circ (i+2, j)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 =$ the Go-board of $f \circ (i+1, j)$ but $f_2 =$ the Go-board of $f \circ (i, j)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i+2, j)$ or $f_2 =$ the Go-board of $f \circ (i+2, j)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i, j)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+2, j)$ or $f_k =$ the Go-board of $f \circ (i+2, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(59) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \text{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq \text{width}$ the Go-board of f,

(v) $\mathcal{L}$(the Go-board of $f \circ (i, j)$, the Go-board of $f \circ (i+1, j)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}$(the Go-board of $f \circ (i+1, j)$, the Go-board of $f \circ (i+1, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 =$ the Go-board of $f \circ (i+1, j)$ but $f_2 =$ the Go-board of $f \circ (i, j)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i+1, j+1)$ or $f_2 =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i, j)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$ and $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+1)$ or $f_k =$ the Go-board of $f \circ (i+1, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j)$.

(60) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \text{len the Go-board of } f$,

(iii) $1 \leq j$,

(iv) $j+1 \leq \text{width the Go-board of } f$,

(v) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j+1), \text{the Go-board of } f \circ (i, j+1)) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $f_1 = \text{the Go-board of } f \circ (i+1, j+1)$ but $f_2 = \text{the Go-board of } f \circ (i+1, j)$ and $f_{\text{len} f -' 1} =$ the Go-board of $f \circ (i, j+1)$ or $f_2 = \text{the Go-board of } f \circ (i, j+1)$ and $f_{\text{len} f -' 1} = \text{the Go-board of } f \circ (i+1, j)$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $f_{k+1} = \text{the Go-board of } f \circ (i+1, j+1)$ and $f_k = \text{the Go-board of } f \circ (i+1, j)$ and $f_{k+2} = \text{the Go-board of } f \circ (i, j+1)$ or $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} = \text{the Go-board of } f \circ (i+1, j)$.

(61) Suppose $1 \leq i$ and $i < \text{len the Go-board of } f$ and $1 \leq j$ and $j+1 < \text{width the Go-board of } f$. Then

(i) $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(ii) $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i, j+2)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(iii) $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(62) Suppose $1 \leq i$ and $i < \text{len the Go-board of } f$ and $1 \leq j$ and $j+1 < \text{width the Go-board of } f$. Then

(i) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(ii) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j+1), \text{the Go-board of } f \circ (i+1, j+2)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(iii) $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(63) Suppose $1 \leq j$ and $j < \text{width the Go-board of } f$ and $1 \leq i$ and $i+1 < \text{len the Go-board of } f$. Then

(i) $\mathcal{L}(\text{the Go-board of } f \circ (i, j), \text{the Go-board of } f \circ (i+1, j)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(ii) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+2, j)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(iii) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(64) Suppose $1 \leq j$ and $j < \text{width the Go-board of } f$ and $1 \leq i$ and $i+1 < \text{len the Go-board of } f$. Then

(i) $\mathcal{L}(\text{the Go-board of } f \circ (i, j+1), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(ii) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j+1), \text{the Go-board of } f \circ (i+2, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(iii) $\mathcal{L}(\text{the Go-board of } f \circ (i+1, j), \text{the Go-board of } f \circ (i+1, j+1)) \not\subseteq \widetilde{\mathcal{L}}(f)$.

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Received October 15, 1995

Published January 2, 2004