JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
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Correctness of Binary Counter Circuits

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Summary. This article introduces the verification of the correctness for the operations and the specification of the 3-bit counter. Both cases: without reset input and with reset input are considered. The proof was proposed by Y. Nakamura in [1].

MML Identifier: GATE_2.

WWW: http://mizar.org/JFM/Vol11/gate_2.html

The article [1] provides the notation and terminology for this paper.

In this paper a, b, c, d denote sets.

We now state four propositions:

(1) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, q_1, q_2, q_3, n_1, n_2, n_3 be sets such that NE s_0 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,q_3, \mathrm{NOT1}\,q_2, \mathrm{NOT1}\,q_1)$ and NE s_1 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,q_3, \mathrm{NOT1}\,q_2, q_1)$ and NE s_2 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,q_3, q_2, \mathrm{NOT1}\,q_1)$ and NE s_3 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,q_3, q_2, q_1)$ and NE s_4 iff NE $\mathrm{AND3}(q_3, \mathrm{NOT1}\,q_2, \mathrm{NOT1}\,q_1)$ and NE s_5 iff NE $\mathrm{AND3}(q_3, \mathrm{NOT1}\,q_2, q_1)$ and NE s_6 iff NE $\mathrm{AND3}(q_3, q_2, \mathrm{NOT1}\,q_1)$ and NE s_7 iff NE $\mathrm{AND3}(q_3, q_2, q_1)$ and NE n_0 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,n_3, \mathrm{NOT1}\,n_2, \mathrm{NOT1}\,n_1)$ and NE n_4 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,n_3, \mathrm{NOT1}\,n_2, n_1)$ and NE n_5 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,n_3, n_2, \mathrm{NOT1}\,n_1)$ and NE n_6 iff NE $\mathrm{AND3}(\mathrm{NOT1}\,n_3, n_2, n_1)$ and NE n_7 iff NE $\mathrm{AND3}(n_3, \mathrm{NOT1}\,n_2, \mathrm{NOT1}\,n_1)$ and NE n_8 iff NE $\mathrm{AND3}(n_3, \mathrm{NOT1}\,n_2, n_1)$ and NE n_9 iff NE $\mathrm{AND3}(n_3, n_2, \mathrm{NOT1}\,n_1)$ and NE n_{10} iff NE $\mathrm{AND3}(n_3, n_2, n_1)$ and NE n_1 iff NE $\mathrm{NOT1}\,q_1$ and NE n_2 iff NE $\mathrm{XOR2}(q_1, q_2)$ and NE n_3 iff NE $\mathrm{OR2}(\mathrm{AND2}(q_3, \mathrm{NOT1}\,q_1), \mathrm{AND2}(q_1, \mathrm{XOR2}(q_2, q_3)))$. Then

(i) NE n_4 iff NE s_0,

(ii) NE n_5 iff NE s_1,

(iii) NE n_6 iff NE s_2,

(iv) NE n_7 iff NE s_3,

(v) NE n_8 iff NE s_4,

(vi) NE n_9 iff NE s_5,

(vii) NE n_{10} iff NE s_6, and

(viii) NE n_0 iff NE s_7.

(2) NE $\mathrm{AND3}(\mathrm{AND2}(a, d), \mathrm{AND2}(b, d), \mathrm{AND2}(c, d))$ iff NE $\mathrm{AND2}(\mathrm{AND3}(a, b, c), d)$.

(3)(i) Not NE AND2(a,b) iff NE OR2(NOT1 a, NOT1 b),

(ii) NE OR2(a,b) and NE OR2(c,b) iff NE OR2(AND2(a,c), b),

(iii) NE OR2(a,b) and NE OR2(c,b) and NE OR2(d,b) iff NE OR2(AND3(a,c,d), b), and

(iv) if NE OR2(a,b) and NE a iff NE c, then NE OR2(c,b).

(4) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, q_1, q_2, q_3, n_1, n_2, n_3, R be sets such that NE s_0 iff NE AND3(NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND3(NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND3(NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND3(NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND3(q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND3(q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND3(q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND3(q_3,q_2,q_1) and NE n_0 iff NE AND3(NOT1 n_3, NOT1 n_2, NOT1 n_1) and NE n_4 iff NE AND3(NOT1 n_3, NOT1 n_2, n_1) and NE n_5 iff NE AND3(NOT1 n_3, n_2, NOT1 n_1) and NE n_6 iff NE AND3(NOT1 n_3, n_2, n_1) and NE n_7 iff NE AND3(n_3, NOT1 n_2, NOT1 n_1) and NE n_8 iff NE AND3(n_3, NOT1 n_2, n_1) and NE n_9 iff NE AND3(n_3, n_2, NOT1 n_1) and NE n_{10} iff NE AND3(n_3,n_2,n_1) and NE n_1 iff NE AND2(NOT1 q_1, R) and NE n_2 iff NE AND2(XOR2(q_1,q_2), R) and NE n_3 iff NE AND2(OR2(AND2(q_3, NOT1 q_1), AND2(q_1, XOR2(q_2,q_3))), R). Then

(i) NE n_4 iff NE AND2(s_0,R),

(ii) NE n_5 iff NE AND2(s_1,R),

(iii) NE n_6 iff NE AND2(s_2,R),

(iv) NE n_7 iff NE AND2(s_3,R),

(v) NE n_8 iff NE AND2(s_4,R),

(vi) NE n_9 iff NE AND2(s_5,R),

(vii) NE n_{10} iff NE AND2(s_6,R), and

(viii) NE n_0 iff NE OR2(s_7, NOT1 R).

REFERENCES

[1] Yatsuka Nakamura. Logic gates and logical equivalence of adders. *Journal of Formalized Mathematics*, 11, 1999. http://mizar.org/JFM/Vol11/gate_1.html.

Received March 13, 1999

Published January 2, 2004