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Boolean Domains[1]

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Summary. BOOLE DOMAIN is a SET DOMAIN that is closed under union and difference. This condition is equivalent to being closed under symmetric difference and one of the following operations: union, intersection or difference. We introduce the set of all finite subsets of a set *A*, denoted by Fin *A*. The mode Finite Subset of a set *A* is introduced with the mother type: Element of Fin *A*. In consequence, "Finite Subset of ..." is an elementary type, therefore one may use such types as "set of Finite Subset of *A*", "[(Finite Subset of *A*), Finite Subset of *A*]", and so on. The article begins with some auxiliary theorems that belong really to [3] or [1] but are missing there. Moreover, bool *A* is redefined as a SET DOMAIN, for an arbitrary set *A*.

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The articles [5], [2], [6], and [4] provide the notation and terminology for this paper.

In this paper X, Y denote sets.

Let I_1 be a set. We say that I_1 is $\cup$-closed if and only if:

(Def. 1) For all sets X, Y such that $X \in I_1$ and $Y \in I_1$ holds $X \cup Y \in I_1$.

We say that I_1 is $\cap$-closed if and only if:

(Def. 2) For all sets X, Y such that $X \in I_1$ and $Y \in I_1$ holds $X \cap Y \in I_1$.

We say that I_1 is diff-closed if and only if:

(Def. 3) For all sets X, Y such that $X \in I_1$ and $Y \in I_1$ holds $X \setminus Y \in I_1$.

Let I_1 be a set. We say that I_1 is preboolean if and only if:

(Def. 4) I_1 is $\cup$-closed and diff-closed.

One can check that every set which is preboolean is also $\cup$-closed and diff-closed and every set which is $\cup$-closed and diff-closed is also preboolean.

Let us note that there exists a set which is non empty, $\cup$-closed, $\cap$-closed, and diff-closed.

In the sequel A is a non empty preboolean set.

The following proposition is true

(10)[1] Let A be a set. Then A is preboolean if and only if for all sets X, Y such that $X \in A$ and $Y \in A$ holds $X \cup Y \in A$ and $X \setminus Y \in A$.

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[1] The propositions (1)–(9) have been removed.

Let us consider A and let X, Y be elements of A. Then $X \cup Y$ is an element of A. Then $X \setminus Y$ is an element of A.

The following propositions are true:

(13)[2] If X is an element of A and Y is an element of A, then $X \cap Y$ is an element of A.

(14) If X is an element of A and Y is an element of A, then $X \dot{-} Y$ is an element of A.

(15) For every non empty set A such that for all elements X, Y of A holds $X \dot{-} Y \in A$ and $X \setminus Y \in A$ holds A is preboolean.

(16) For every non empty set A such that for all elements X, Y of A holds $X \dot{-} Y \in A$ and $X \cap Y \in A$ holds A is preboolean.

(17) For every non empty set A such that for all elements X, Y of A holds $X \dot{-} Y \in A$ and $X \cup Y \in A$ holds A is preboolean.

Let us consider A and let X, Y be elements of A. Then $X \cap Y$ is an element of A. Then $X \dot{-} Y$ is an element of A.

Next we state three propositions:

(18) $\emptyset \in A$.

(20)[3] For every set A holds 2^A is preboolean.

(21) For all non empty preboolean sets A, B holds $A \cap B$ is non empty and preboolean.

In the sequel A, B denote sets.

Let us consider A. The functor $\mathrm{Fin}\, A$ yields a preboolean set and is defined by:

(Def. 5) For every set X holds $X \in \mathrm{Fin}\, A$ iff $X \subseteq A$ and X is finite.

Let us consider A. Note that $\mathrm{Fin}\, A$ is non empty.

Let us consider A. Note that every element of $\mathrm{Fin}\, A$ is finite.

We now state several propositions:

(23)[4] If $A \subseteq B$, then $\mathrm{Fin}\, A \subseteq \mathrm{Fin}\, B$.

(24) $\mathrm{Fin}(A \cap B) = \mathrm{Fin}\, A \cap \mathrm{Fin}\, B$.

(25) $\mathrm{Fin}\, A \cup \mathrm{Fin}\, B \subseteq \mathrm{Fin}(A \cup B)$.

(26) $\mathrm{Fin}\, A \subseteq 2^A$.

(27) If A is finite, then $\mathrm{Fin}\, A = 2^A$.

(28) $\mathrm{Fin}\, \emptyset = \{\emptyset\}$.

Let us consider A. A finite subset of A is an element of $\mathrm{Fin}\, A$.

We now state three propositions:

(30)[5] Every finite subset of A is finite.

(32)[6] Every finite subset of A is a subset of A.

(34)[7] If A is finite, then every subset of A is a finite subset of A.

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[2] The propositions (11) and (12) have been removed.
[3] The proposition (19) has been removed.
[4] The proposition (22) has been removed.
[5] The proposition (29) has been removed.
[6] The proposition (31) has been removed.
[7] The proposition (33) has been removed.

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