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Non-contiguous Substrings and One-to-one Finite Sequences

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Summary. This text is a continuation of [3]. We prove a number of theorems concerning both notions introduced there and one-to-one finite sequences. We introduce a function that removes from a string elements of the string that belongs to a given set.

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The articles [8], [7], [10], [9], [3], [11], [4], [5], [6], [1], and [2] provide the notation and terminology for this paper.

We adopt the following rules: p, q, r are finite sequences, u, v, x, y, z, A, D, X, Y are sets, and i, j, k, l, m, n are natural numbers.

One can prove the following propositions:

(1) $\mathrm{Seg}\,3 = \{1,2,3\}$.

(2) $\mathrm{Seg}\,4 = \{1,2,3,4\}$.

(3) $\mathrm{Seg}\,5 = \{1,2,3,4,5\}$.

(4) $\mathrm{Seg}\,6 = \{1,2,3,4,5,6\}$.

(5) $\mathrm{Seg}\,7 = \{1,2,3,4,5,6,7\}$.

(6) $\mathrm{Seg}\,8 = \{1,2,3,4,5,6,7,8\}$.

(7) $\mathrm{Seg}\,k = \emptyset$ iff $k \notin \mathrm{Seg}\,k$.

(9)[1] $k+1 \notin \mathrm{Seg}\,k$.

(10) If $k \neq 0$, then $k \in \mathrm{Seg}(k+n)$.

(11) If $k+n \in \mathrm{Seg}\,k$, then $n = 0$.

(12) If $k < n$, then $k+1 \in \mathrm{Seg}\,n$.

(13) If $k \in \mathrm{Seg}\,n$ and $m < k$, then $k-m \in \mathrm{Seg}\,n$.

(14) $k-n \in \mathrm{Seg}\,k$ iff $n < k$.

(15) $\mathrm{Seg}\,k$ misses $\{k+1\}$.

(16) $\mathrm{Seg}(k+1) \setminus \mathrm{Seg}\,k = \{k+1\}$.

[1] The proposition (8) has been removed.

(17) $\operatorname{Seg} k \neq \operatorname{Seg}(k+1)$.

(18) If $\operatorname{Seg} k = \operatorname{Seg}(k+n)$, then $n = 0$.

(19) $\operatorname{Seg} k \subseteq \operatorname{Seg}(k+n)$.

(20) $\operatorname{Seg} k$ and $\operatorname{Seg} n$ are $\subseteq$-comparable.

(22)[2] If $\operatorname{Seg} k = \{y\}$, then $k = 1$ and $y = 1$.

(23) If $\operatorname{Seg} k = \{x, y\}$ and $x \neq y$, then $k = 2$ and $\{x, y\} = \{1, 2\}$.

(24) If $x \in \operatorname{dom} p$, then $x \in \operatorname{dom}(p \frown q)$.

(25) If $x \in \operatorname{dom} p$, then x is a natural number.

(26) If $x \in \operatorname{dom} p$, then $x \neq 0$.

(27) $n \in \operatorname{dom} p$ iff $1 \leq n$ and $n \leq \operatorname{len} p$.

(28) $n \in \operatorname{dom} p$ iff $n - 1$ is a natural number and $\operatorname{len} p - n$ is a natural number.

(29) $\operatorname{dom}\langle x, y\rangle = \operatorname{Seg} 2$.

(30) $\operatorname{dom}\langle x, y, z\rangle = \operatorname{Seg} 3$.

(31) $\operatorname{len} p = \operatorname{len} q$ iff $\operatorname{dom} p = \operatorname{dom} q$.

(32) $\operatorname{len} p \leq \operatorname{len} q$ iff $\operatorname{dom} p \subseteq \operatorname{dom} q$.

(33) If $x \in \operatorname{rng} p$, then $1 \in \operatorname{dom} p$.

(34) If $\operatorname{rng} p \neq \emptyset$, then $1 \in \operatorname{dom} p$.

(38)[3] $\emptyset \neq \langle x, y\rangle$.

(39) $\emptyset \neq \langle x, y, z\rangle$.

(40) $\langle x\rangle \neq \langle y, z\rangle$.

(41) $\langle u\rangle \neq \langle x, y, z\rangle$.

(42) $\langle u, v\rangle \neq \langle x, y, z\rangle$.

(43) If $\operatorname{len} r = \operatorname{len} p + \operatorname{len} q$ and for every k such that $k \in \operatorname{dom} p$ holds $r(k) = p(k)$ and for every k such that $k \in \operatorname{dom} q$ holds $r(\operatorname{len} p + k) = q(k)$, then $r = p \frown q$.

(44) For every finite set A such that $A \subseteq \operatorname{Seg} k$ holds $\operatorname{len}\operatorname{Sgm} A = \operatorname{card} A$.

(45) For every finite set A such that $A \subseteq \operatorname{Seg} k$ holds $\operatorname{dom}\operatorname{Sgm} A = \operatorname{Seg}\operatorname{card} A$.

(46) If $X \subseteq \operatorname{Seg} i$ and $k < l$ and $1 \leq n$ and $m \leq \operatorname{len}\operatorname{Sgm} X$ and $(\operatorname{Sgm} X)(m) = k$ and $(\operatorname{Sgm} X)(n) = l$, then $m < n$.

(48)[4] If $X \subseteq \operatorname{Seg} i$ and $Y \subseteq \operatorname{Seg} j$, then for all m, n such that $m \in X$ and $n \in Y$ holds $m < n$ iff $\operatorname{Sgm}(X \cup Y) = (\operatorname{Sgm} X) \frown \operatorname{Sgm} Y$.

(49) $\operatorname{Sgm} \emptyset = \emptyset$.

(50) If $0 \neq n$, then $\operatorname{Sgm}\{n\} = \langle n\rangle$.

(51) If $0 < n$ and $n < m$, then $\operatorname{Sgm}\{n, m\} = \langle n, m\rangle$.

[2] The proposition (21) has been removed.
[3] The propositions (35)–(37) have been removed.
[4] The proposition (47) has been removed.

(52) $\operatorname{len}\operatorname{Sgm}\operatorname{Seg}k = k$.

(53) $\operatorname{Sgm}\operatorname{Seg}(k+n) \restriction \operatorname{Seg}k = \operatorname{Sgm}\operatorname{Seg}k$.

(54) $\operatorname{Sgm}\operatorname{Seg}k = \operatorname{idseq}(k)$.

(55) $p \restriction \operatorname{Seg}n = p$ iff $\operatorname{len}p \leq n$.

(56) $\operatorname{idseq}(n+k) \restriction \operatorname{Seg}n = \operatorname{idseq}(n)$.

(57) $\operatorname{idseq}(n) \restriction \operatorname{Seg}m = \operatorname{idseq}(m)$ iff $m \leq n$.

(58) $\operatorname{idseq}(n) \restriction \operatorname{Seg}m = \operatorname{idseq}(n)$ iff $n \leq m$.

(59) If $\operatorname{len}p = k+l$ and $q = p \restriction \operatorname{Seg}k$, then $\operatorname{len}q = k$.

(60) If $\operatorname{len}p = k+l$ and $q = p \restriction \operatorname{Seg}k$, then $\operatorname{dom}q = \operatorname{Seg}k$.

(61) If $\operatorname{len}p = k+1$ and $q = p \restriction \operatorname{Seg}k$, then $p = q \frown \langle p(k+1) \rangle$.

(62) $p \restriction X$ is a finite sequence iff there exists k such that $X \cap \operatorname{dom}p = \operatorname{Seg}k$.

Let p be a finite sequence and let A be a set. Observe that $p^{-1}(A)$ is finite.
We now state two propositions:

(63) $\operatorname{card}((p \frown q)^{-1}(A)) = \operatorname{card}(p^{-1}(A)) + \operatorname{card}(q^{-1}(A))$.

(64) $p^{-1}(A) \subseteq (p \frown q)^{-1}(A)$.

Let us consider p, A. The functor $p - A$ yielding a finite sequence is defined by:

(Def. 1) $p - A = p \cdot \operatorname{Sgm}(\operatorname{dom}p \setminus p^{-1}(A))$.

The following propositions are true:

(66)[5] $\operatorname{len}(p - A) = \operatorname{len}p - \operatorname{card}(p^{-1}(A))$.

(67) $\operatorname{len}(p - A) \leq \operatorname{len}p$.

(68) If $\operatorname{len}(p - A) = \operatorname{len}p$, then A misses $\operatorname{rng}p$.

(69) If $n = \operatorname{len}p - \operatorname{card}(p^{-1}(A))$, then $\operatorname{dom}(p - A) = \operatorname{Seg}n$.

(70) $\operatorname{dom}(p - A) \subseteq \operatorname{dom}p$.

(71) If $\operatorname{dom}(p - A) = \operatorname{dom}p$, then A misses $\operatorname{rng}p$.

(72) $\operatorname{rng}(p - A) = \operatorname{rng}p \setminus A$.

(73) $\operatorname{rng}(p - A) \subseteq \operatorname{rng}p$.

(74) If $\operatorname{rng}(p - A) = \operatorname{rng}p$, then A misses $\operatorname{rng}p$.

(75) $p - A = \emptyset$ iff $\operatorname{rng}p \subseteq A$.

(76) $p - A = p$ iff A misses $\operatorname{rng}p$.

(77) $p - \{x\} = p$ iff $x \notin \operatorname{rng}p$.

(78) $p - \emptyset = p$.

(79) $p - \operatorname{rng}p = \emptyset$.

(80) $p \frown q - A = (p - A) \frown (q - A)$.

[5] The proposition (65) has been removed.

(81) $\emptyset - A = \emptyset$.

(82) $\langle x \rangle - A = \langle x \rangle$ iff $x \notin A$.

(83) $\langle x \rangle - A = \emptyset$ iff $x \in A$.

(84) $\langle x, y \rangle - A = \emptyset$ iff $x \in A$ and $y \in A$.

(85) If $x \in A$ and $y \notin A$, then $\langle x, y \rangle - A = \langle y \rangle$.

(86) If $\langle x, y \rangle - A = \langle y \rangle$ and $x \neq y$, then $x \in A$ and $y \notin A$.

(87) If $x \notin A$ and $y \in A$, then $\langle x, y \rangle - A = \langle x \rangle$.

(88) If $\langle x, y \rangle - A = \langle x \rangle$ and $x \neq y$, then $x \notin A$ and $y \in A$.

(89) $\langle x, y \rangle - A = \langle x, y \rangle$ iff $x \notin A$ and $y \notin A$.

(90) If $\operatorname{len} p = k + 1$ and $q = p \restriction \operatorname{Seg} k$, then $p(k+1) \in A$ iff $p - A = q - A$.

(91) If $\operatorname{len} p = k + 1$ and $q = p \restriction \operatorname{Seg} k$, then $p(k+1) \notin A$ iff $p - A = (q - A) ^\frown \langle p(k+1) \rangle$.

(92) If $n \in \operatorname{dom} p$, then for every finite set B such that $B = \{k : k \in \operatorname{dom} p \land k \leq n \land p(k) \in A\}$ holds $p(n) \in A$ or $(p - A)(n - \operatorname{card} B) = p(n)$.

(93) If p is a finite sequence of elements of D, then $p - A$ is a finite sequence of elements of D.

(94) If p is one-to-one, then $p - A$ is one-to-one.

(95) If p is one-to-one, then $\operatorname{len}(p - A) = \operatorname{len} p - \operatorname{card}(A \cap \operatorname{rng} p)$.

(96) For every finite set A such that p is one-to-one and $A \subseteq \operatorname{rng} p$ holds $\operatorname{len}(p - A) = \operatorname{len} p - \operatorname{card} A$.

(97) If p is one-to-one and $x \in \operatorname{rng} p$, then $\operatorname{len}(p - \{x\}) = \operatorname{len} p - 1$.

(98) $\operatorname{rng} p$ misses $\operatorname{rng} q$ and p is one-to-one and q is one-to-one iff $p ^\frown q$ is one-to-one.

(99) If $A \subseteq \operatorname{Seg} k$, then $\operatorname{Sgm} A$ is one-to-one.

(100) $\operatorname{idseq}(n)$ is one-to-one.

(101) $\emptyset$ is one-to-one.

(102) $\langle x \rangle$ is one-to-one.

(103) $x \neq y$ iff $\langle x, y \rangle$ is one-to-one.

(104) $x \neq y$ and $y \neq z$ and $z \neq x$ iff $\langle x, y, z \rangle$ is one-to-one.

(105) If p is one-to-one and $\operatorname{rng} p = \{x\}$, then $\operatorname{len} p = 1$.

(106) If p is one-to-one and $\operatorname{rng} p = \{x\}$, then $p = \langle x \rangle$.

(107) If p is one-to-one and $\operatorname{rng} p = \{x, y\}$ and $x \neq y$, then $\operatorname{len} p = 2$.

(108) If p is one-to-one and $\operatorname{rng} p = \{x, y\}$ and $x \neq y$, then $p = \langle x, y \rangle$ or $p = \langle y, x \rangle$.

(109) If p is one-to-one and $\operatorname{rng} p = \{x, y, z\}$ and $\langle x, y, z \rangle$ is one-to-one, then $\operatorname{len} p = 3$.

(110) If p is one-to-one and $\operatorname{rng} p = \{x, y, z\}$ and $x \neq y$ and $y \neq z$ and $x \neq z$, then $\operatorname{len} p = 3$.

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