JOURNAL OF FORMALIZED MATHEMATICS
Volume 6, Released 1994, Published 2003
Inst. of Computer Science, Univ. of Białystok

On the Decomposition of the Continuity

Marian Przemski
Warsaw University
Białystok

Summary. This article is devoted to functions of general topological spaces. A function from X to Y is A-continuous if the counterimage of every open set V of Y belongs to A, where A is a collection of subsets of X. We give the following characteristics of the continuity, called decomposition of continuity: A function f is continuous if and only if it is both A-continuous and B-continuous.

MML Identifier: DECOMP_1.

WWW: http://mizar.org/JFM/Vol6/decomp_1.html

The articles [3], [1], [2], and [4] provide the notation and terminology for this paper.

Let T be a non empty topological space. A subset of T is called an α-set of T if:

(Def. 1) $\mathrm{It} \subseteq \mathrm{Int}\,\overline{\mathrm{Int}\,\mathrm{it}}$.

Let I_1 be a subset of T. We say that I_1 is semi-open if and only if:

(Def. 2) $I_1 \subseteq \overline{\mathrm{Int}\,I_1}$.

We say that I_1 is pre-open if and only if:

(Def. 3) $I_1 \subseteq \mathrm{Int}\,\overline{I_1}$.

We say that I_1 is pre-semi-open if and only if:

(Def. 4) $I_1 \subseteq \overline{\mathrm{Int}\,\overline{I_1}}$.

We say that I_1 is semi-pre-open if and only if:

(Def. 5) $I_1 \subseteq \overline{\mathrm{Int}\,I_1} \cup \mathrm{Int}\,\overline{I_1}$.

Let T be a non empty topological space and let B be a subset of T. The functor $\mathrm{sInt}(B)$ yielding a subset of T is defined as follows:

(Def. 6) $\mathrm{sInt}(B) = B \cap \overline{\mathrm{Int}\,B}$.

The functor $\mathrm{pInt}(B)$ yielding a subset of T is defined by:

(Def. 7) $\mathrm{pInt}(B) = B \cap \mathrm{Int}\,\overline{B}$.

The functor $\alpha\mathrm{Int}(B)$ yielding a subset of T is defined as follows:

(Def. 8) $\alpha\mathrm{Int}(B) = B \cap \mathrm{Int}\,\overline{\mathrm{Int}\,B}$.

The functor $\mathrm{psInt}(B)$ yielding a subset of T is defined by:

(Def. 9) $\mathrm{psInt}(B) = B \cap \overline{\mathrm{Int}\overline{B}}$.

Let T be a non empty topological space and let B be a subset of T. The functor $\mathrm{spInt}(B)$ yields a subset of T and is defined as follows:

(Def. 10) $\mathrm{spInt}(B) = \mathrm{sInt}(B) \cup \mathrm{pInt}(B)$.

Let T be a non empty topological space. The functor T^{α} yields a family of subsets of T and is defined by:

(Def. 11) $T^{\alpha} = \{B; B$ ranges over subsets of T: B is an α-set of $T\}$.

The functor $\mathrm{SO}(T)$ yielding a family of subsets of T is defined by:

(Def. 12) $\mathrm{SO}(T) = \{B; B$ ranges over subsets of T: B is semi-open$\}$.

The functor $\mathrm{PO}(T)$ yielding a family of subsets of T is defined by:

(Def. 13) $\mathrm{PO}(T) = \{B; B$ ranges over subsets of T: B is pre-open$\}$.

The functor $\mathrm{SPO}(T)$ yielding a family of subsets of T is defined as follows:

(Def. 14) $\mathrm{SPO}(T) = \{B; B$ ranges over subsets of T: B is semi-pre-open$\}$.

The functor $\mathrm{PSO}(T)$ yields a family of subsets of T and is defined by:

(Def. 15) $\mathrm{PSO}(T) = \{B; B$ ranges over subsets of T: B is pre-semi-open$\}$.

The functor $D(c,\alpha)(T)$ yielding a family of subsets of T is defined by:

(Def. 16) $D(c,\alpha)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \alpha\mathrm{Int}(B)\}$.

The functor $D(c,p)(T)$ yielding a family of subsets of T is defined by:

(Def. 17) $D(c,p)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{pInt}(B)\}$.

The functor $D(c,s)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 18) $D(c,s)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{sInt}(B)\}$.

The functor $D(c,ps)(T)$ yielding a family of subsets of T is defined by:

(Def. 19) $D(c,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{psInt}(B)\}$.

The functor $D(\alpha,p)(T)$ yields a family of subsets of T and is defined by:

(Def. 20) $D(\alpha,p)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)\}$.

The functor $D(\alpha,s)(T)$ yields a family of subsets of T and is defined by:

(Def. 21) $D(\alpha,s)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{sInt}(B)\}$.

The functor $D(\alpha,ps)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 22) $D(\alpha,ps)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{psInt}(B)\}$.

The functor $D(p,sp)(T)$ yields a family of subsets of T and is defined by:

(Def. 23) $D(p,sp)(T) = \{B; B$ ranges over subsets of T: $\mathrm{pInt}(B) = \mathrm{spInt}(B)\}$.

The functor $D(p,ps)(T)$ yields a family of subsets of T and is defined by:

(Def. 24) $D(p,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{pInt}(B) = \mathrm{psInt}(B)\}$.

The functor $D(sp,ps)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 25) $D(sp,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{spInt}(B) = \mathrm{psInt}(B)\}$.

In the sequel T denotes a non empty topological space and B denotes a subset of T.
The following propositions are true:

(1) $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)$ iff $\mathrm{sInt}(B) = \mathrm{psInt}(B)$.

(2) B is an α-set of T iff $B = \alpha\mathrm{Int}(B)$.

(3) B is semi-open iff $B = \mathrm{sInt}(B)$.

(4) B is pre-open iff $B = \mathrm{pInt}(B)$.

(5) B is pre-semi-open iff $B = \mathrm{psInt}(B)$.

(6) B is semi-pre-open iff $B = \mathrm{spInt}(B)$.

(7) $T^{\alpha} \cap D(c,\alpha)(T) =$ the topology of T.

(8) $\mathrm{SO}(T) \cap D(c,s)(T) =$ the topology of T.

(9) $\mathrm{PO}(T) \cap D(c,p)(T) =$ the topology of T.

(10) $\mathrm{PSO}(T) \cap D(c,ps)(T) =$ the topology of T.

(11) $\mathrm{PO}(T) \cap D(\alpha,p)(T) = T^{\alpha}$.

(12) $\mathrm{SO}(T) \cap D(\alpha,s)(T) = T^{\alpha}$.

(13) $\mathrm{PSO}(T) \cap D(\alpha,ps)(T) = T^{\alpha}$.

(14) $\mathrm{SPO}(T) \cap D(p,sp)(T) = \mathrm{PO}(T)$.

(15) $\mathrm{PSO}(T) \cap D(p,ps)(T) = \mathrm{PO}(T)$.

(16) $\mathrm{PSO}(T) \cap D(\alpha,p)(T) = \mathrm{SO}(T)$.

(17) $\mathrm{PSO}(T) \cap D(sp,ps)(T) = \mathrm{SPO}(T)$.

Let X, Y be non empty topological spaces and let f be a map from X into Y. We say that f is s-continuous if and only if:

(Def. 26) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{SO}(X)$.

We say that f is p-continuous if and only if:

(Def. 27) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{PO}(X)$.

We say that f is α-continuous if and only if:

(Def. 28) For every subset G of Y such that G is open holds $f^{-1}(G) \in X^{\alpha}$.

We say that f is ps-continuous if and only if:

(Def. 29) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{PSO}(X)$.

We say that f is sp-continuous if and only if:

(Def. 30) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{SPO}(X)$.

We say that f is (c,α)-continuous if and only if:

(Def. 31) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,\alpha)(X)$.

We say that f is (c,s)-continuous if and only if:

(Def. 32) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,s)(X)$.

We say that f is (c,p)-continuous if and only if:

(Def. 33) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,p)(X)$.

We say that f is (c,ps)-continuous if and only if:

(Def. 34) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,ps)(X)$.

We say that f is (α,p)-continuous if and only if:

(Def. 35) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,p)(X)$.

We say that f is (α,s)-continuous if and only if:

(Def. 36) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,s)(X)$.

We say that f is (α,ps)-continuous if and only if:

(Def. 37) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,ps)(X)$.

We say that f is (p,ps)-continuous if and only if:

(Def. 38) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(p,ps)(X)$.

We say that f is (p,sp)-continuous if and only if:

(Def. 39) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(p,sp)(X)$.

We say that f is (sp,ps)-continuous if and only if:

(Def. 40) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(sp,ps)(X)$.

In the sequel X, Y denote non empty topological spaces and f denotes a map from X into Y. One can prove the following propositions:

(18) f is α-continuous iff f is p-continuous and (α,p)-continuous.

(19) f is α-continuous iff f is s-continuous and (α,s)-continuous.

(20) f is α-continuous iff f is ps-continuous and (α,ps)-continuous.

(21) f is p-continuous iff f is sp-continuous and (p,sp)-continuous.

(22) f is p-continuous iff f is ps-continuous and (p,ps)-continuous.

(23) f is s-continuous iff f is ps-continuous and (α,p)-continuous.

(24) f is sp-continuous iff f is ps-continuous and (sp,ps)-continuous.

(25) f is continuous iff f is α-continuous and (c,α)-continuous.

(26) f is continuous iff f is s-continuous and (c,s)-continuous.

(27) f is continuous iff f is p-continuous and (c,p)-continuous.

(28) f is continuous iff f is ps-continuous and (c,ps)-continuous.

Acknowledgments

The author wishes to thank Professor A. Trybulec for many helpful talks during the preparation of this paper.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[2] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[3] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[4] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/tops_1.html.

Received December 12, 1994

Published January 2, 2004