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Shear Theorems and Their Role in Affine Geometry

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Summary. Investigations on affine shear theorems, major and minor, direct and indirect. We prove logical relationships which hold between these statements and between them and other classical affine configurational axioms (eg. minor and major Pappus Axiom, Desargues Axioms et al.). For the shear, Desargues, and Pappus Axioms formulated in terms of metric affine spaces we prove they are equivalent to corresponding statements formulated in terms of affine reduct of the given space.

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The articles [8], [2], [4], [5], [1], [3], [6], and [7] provide the notation and terminology for this paper.

We use the following convention: X denotes an affine plane, o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 denote elements of X, and M, N denote subsets of X.

Let us consider X. We say that X satisfies minor Scherungssatz if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that $M \parallel N$ and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_2 \notin M$ and $b_4 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_1 \notin N$ and $b_3 \notin N$ and $a_3, a_2 \upuparrows b_3, b_2$ and $a_2, a_1 \upuparrows b_2, b_1$ and $a_1, a_4 \upuparrows b_1, b_4$. Then $a_3, a_4 \upuparrows b_3, b_4$.

We introduce X satisfies minor Scherungssatz as a synonym of X satisfies minor Scherungssatz.

Let us consider X. We say that X satisfies major Scherungssatz if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let given o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that M is a line and N is a line and $o \in M$ and $o \in N$ and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_2 \notin M$ and $b_4 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_1 \notin N$ and $b_3 \notin N$ and $a_3, a_2 \upuparrows b_3, b_2$ and $a_2, a_1 \upuparrows b_2, b_1$ and $a_1, a_4 \upuparrows b_1, b_4$. Then $a_3, a_4 \upuparrows b_3, b_4$.

We introduce X satisfies major Scherungssatz as a synonym of X satisfies major Scherungssatz.

Let us consider X. We say that X satisfies Scherungssatz if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that M is a line and N is a line and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_2 \notin M$ and $b_4 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_1 \notin N$ and $b_3 \notin N$ and $a_3, a_2 \upuparrows b_3, b_2$ and $a_2, a_1 \upuparrows b_2, b_1$ and $a_1, a_4 \upuparrows b_1, b_4$. Then $a_3, a_4 \upuparrows b_3, b_4$.

We introduce X satisfies Scherungssatz as a synonym of X satisfies Scherungssatz.

Let us consider X. We say that X satisfies indirect Scherungssatz if and only if the condition (Def. 4) is satisfied.

(Def. 4) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that M is a line and N is a line and $a_1 \in M$ and $a_3 \in M$ and $b_2 \in M$ and $b_4 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_1 \in N$ and $b_3 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_1 \notin M$ and $b_3 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_2 \notin N$ and $b_4 \notin N$ and $a_3, a_2 \upharpoonleft\upharpoonright b_3, b_2$ and $a_2, a_1 \upharpoonleft\upharpoonright b_2, b_1$ and $a_1, a_4 \upharpoonleft\upharpoonright b_1, b_4$. Then $a_3, a_4 \upharpoonleft\upharpoonright b_3, b_4$.

We introduce X satisfies Scherungssatz * as a synonym of X satisfies indirect Scherungssatz.

Let us consider X. We say that X satisfies minor indirect Scherungssatz if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that $M \parallel N$ and $a_1 \in M$ and $a_3 \in M$ and $b_2 \in M$ and $b_4 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_1 \in N$ and $b_3 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_1 \notin M$ and $b_3 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_2 \notin N$ and $b_4 \notin N$ and $a_3, a_2 \upharpoonleft\upharpoonright b_3, b_2$ and $a_2, a_1 \upharpoonleft\upharpoonright b_2, b_1$ and $a_1, a_4 \upharpoonleft\upharpoonright b_1, b_4$. Then $a_3, a_4 \upharpoonleft\upharpoonright b_3, b_4$.

We introduce X satisfies minor Scherungssatz * as a synonym of X satisfies minor indirect Scherungssatz.

Let us consider X. We say that X satisfies major indirect Scherungssatz if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that M is a line and N is a line and $o \in M$ and $o \in N$ and $a_1 \in M$ and $a_3 \in M$ and $b_2 \in M$ and $b_4 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_1 \in N$ and $b_3 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_1 \notin M$ and $b_3 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_2 \notin N$ and $b_4 \notin N$ and $a_3, a_2 \upharpoonleft\upharpoonright b_3, b_2$ and $a_2, a_1 \upharpoonleft\upharpoonright b_2, b_1$ and $a_1, a_4 \upharpoonleft\upharpoonright b_1, b_4$. Then $a_3, a_4 \upharpoonleft\upharpoonright b_3, b_4$.

We introduce X satisfies major Scherungssatz * as a synonym of X satisfies major indirect Scherungssatz.

The following propositions are true:

(1) X satisfies Scherungssatz * if and only if X satisfies minor Scherungssatz * and X satisfies major Scherungssatz *.

(2) X satisfies Scherungssatz if and only if X satisfies minor Scherungssatz and X satisfies major Scherungssatz.

(3) If X satisfies minor Scherungssatz *, then X satisfies minor Scherungssatz.

(4) If X satisfies major Scherungssatz *, then X satisfies major Scherungssatz.

(5) If X satisfies Scherungssatz *, then X satisfies Scherungssatz.

(6) If X satisfies **des**, then X satisfies minor Scherungssatz.

(7) If X satisfies **DES**, then X satisfies major Scherungssatz.

(8) X satisfies **DES** iff X satisfies Scherungssatz.

(9) X satisfies **pap** iff X satisfies minor Scherungssatz *.

(10) X satisfies **PAP** iff X satisfies major Scherungssatz *.

(11) X satisfies **PPAP** iff X satisfies Scherungssatz *.

(12) If X satisfies major Scherungssatz *, then X satisfies minor Scherungssatz *.

In the sequel X denotes a metric affine plane.

The following propositions are true:

(13) The affine reduct of X satisfies Scherungssatz iff Scherungssatz holds in X.

(14) Trapezium variant of Desargues Axiom holds in X if and only if the affine reduct of X satisfies **TDES**.

(15) The affine reduct of X satisfies **des** iff minor Desargues Axiom holds in X.

(16) Pappos Axiom holds in X iff the affine reduct of X satisfies **PAP**.

(17) Desargues Axiom holds in X iff the affine reduct of X satisfies **DES**.

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