JOURNAL OF FORMALIZED MATHEMATICS
Volume 5, Released 1993, Published 2003
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Complex Sequences

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Summary. Definitions of complex sequence and operations on sequences (multiplication of sequences and multiplication by a complex number, addition, subtraction, division and absolute value of sequence) are given. We followed [4].

MML Identifier: COMSEQ_1.

WWW: http://mizar.org/JFM/Vol5/comseq_1.html

The articles [5], [8], [6], [3], [9], [1], [10], [2], [7], and [4] provide the notation and terminology for this paper.

For simplicity, we use the following convention: f is a function, n is a natural number, r, p are elements of $\mathbb{C}$, and x is a set.

A complex sequence is a function from $\mathbb{N}$ into $\mathbb{C}$.

In the sequel s_1, s_2, s_3, s_4, s'_1, s'_2 denote complex sequences.

One can prove the following two propositions:

(1) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every x such that $x \in \mathbb{N}$ holds $f(x)$ is an element of $\mathbb{C}$.

(2) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every n holds $f(n)$ is an element of $\mathbb{C}$.

Let us consider s_1, n. Then $s_1(n)$ is an element of $\mathbb{C}$.

The scheme *ExComplexSeq* deals with a unary functor $\mathcal{F}$ yielding an element of $\mathbb{C}$, and states that:

There exists s_1 such that for every n holds $s_1(n) = \mathcal{F}(n)$

for all values of the parameter.

Let I_1 be a complex sequence. We say that I_1 is non-zero if and only if:

(Def. 1) $\operatorname{rng} I_1 \subseteq \mathbb{C} \setminus \{0_{\mathbb{C}}\}$.

We now state the proposition

(3) s_1 is non-zero iff for every x such that $x \in \mathbb{N}$ holds $s_1(x) \neq 0_{\mathbb{C}}$.

Let us observe that there exists a complex sequence which is non-zero.

The following propositions are true:

(4) s_1 is non-zero iff for every n holds $s_1(n) \neq 0_{\mathbb{C}}$.

(6)[1] For all s_1, s_2 such that for every n holds $s_1(n) = s_2(n)$ holds $s_1 = s_2$.

(7) For every r there exists s_1 such that $\operatorname{rng} s_1 = \{r\}$.

[1] The proposition (5) has been removed.

Let C be a non empty set and let f_1, f_2 be partial functions from C to $\mathbb{C}$. The functor $f_1 + f_2$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 2) $\mathrm{dom}(f_1 + f_2) = \mathrm{dom}\, f_1 \cap \mathrm{dom}\, f_2$ and for every element c of C such that $c \in \mathrm{dom}(f_1 + f_2)$ holds $(f_1 + f_2)(c) = (f_1)_c + (f_2)_c$.

Let us note that the functor $f_1 + f_2$ is commutative. The functor $f_1\ f_2$ yields a partial function from C to $\mathbb{C}$ and is defined by:

(Def. 3) $\mathrm{dom}(f_1\ f_2) = \mathrm{dom}\, f_1 \cap \mathrm{dom}\, f_2$ and for every element c of C such that $c \in \mathrm{dom}(f_1\ f_2)$ holds $(f_1\ f_2)(c) = (f_1)_c \cdot (f_2)_c$.

Let us observe that the functor $f_1\ f_2$ is commutative.

Let C be a non empty set and let f_1, f_2 be functions from C into $\mathbb{C}$. Then $f_1 + f_2$ can be characterized by the condition:

(Def. 4) $\mathrm{dom}(f_1 + f_2) = C$ and for every element c of C holds $(f_1 + f_2)(c) = f_1(c) + f_2(c)$.

Then $f_1\ f_2$ can be characterized by the condition:

(Def. 5) $\mathrm{dom}(f_1\ f_2) = C$ and for every element c of C holds $(f_1\ f_2)(c) = f_1(c) \cdot f_2(c)$.

Let C be a non empty set and let s_2, s_3 be functions from C into $\mathbb{C}$. Note that $s_2 + s_3$ is total and $s_2\ s_3$ is total.

Let C be a non empty set, let f be a partial function from C to $\mathbb{C}$, and let us consider r. The functor $r\ f$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 6) $\mathrm{dom}(r\ f) = \mathrm{dom}\, f$ and for every element c of C such that $c \in \mathrm{dom}(r\ f)$ holds $(r\ f)(c) = r \cdot f_c$.

Let C be a non empty set, let f be a function from C into $\mathbb{C}$, and let us consider r. Then $r\ f$ can be characterized by the condition:

(Def. 7) $\mathrm{dom}(r\ f) = C$ and for every element n of C holds $(r\ f)(n) = r \cdot f(n)$.

Let C be a non empty set, let s_1 be a function from C into $\mathbb{C}$, and let us consider r. Observe that $r\, s_1$ is total.

Let C be a non empty set and let f be a partial function from C to $\mathbb{C}$. The functor $-f$ yielding a partial function from C to $\mathbb{C}$ is defined as follows:

(Def. 8) $\mathrm{dom}(-f) = \mathrm{dom}\, f$ and for every element c of C such that $c \in \mathrm{dom}(-f)$ holds $(-f)(c) = -f_c$.

Let C be a non empty set and let f be a function from C into $\mathbb{C}$. Then $-f$ can be characterized by the condition:

(Def. 9) $\mathrm{dom}(-f) = C$ and for every element n of C holds $(-f)(n) = -f(n)$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Observe that $-s_1$ is total.

Let C be a non empty set and let f_1, f_2 be partial functions from C to $\mathbb{C}$. The functor $f_1 - f_2$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 10) $f_1 - f_2 = f_1 + -f_2$.

Let C be a non empty set and let f_1, f_2 be functions from C into $\mathbb{C}$. Observe that $f_1 - f_2$ is total.

Let us consider s_1. The functor ${s_1}^{-1}$ yielding a complex sequence is defined by:

(Def. 11) For every n holds ${s_1}^{-1}(n) = s_1(n)^{-1}$.

Let us consider s_2, s_1. The functor s_2/s_1 yielding a complex sequence is defined by:

(Def. 12) $s_2/s_1 = s_2\, {s_1}^{-1}$.

Let C be a non empty set and let f be a partial function from C to $\mathbb{C}$. The functor $|f|$ yielding a partial function from C to $\mathbb{R}$ is defined by:

(Def. 13) $\mathrm{dom}\,|f| = \mathrm{dom}\,f$ and for every element c of C such that $c \in \mathrm{dom}\,|f|$ holds $|f|(c) = |f_c|$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Then $|s_1|$ can be characterized by the condition:

(Def. 14) $\mathrm{dom}\,|s_1| = C$ and for every element n of C holds $|s_1|(n) = |s_1(n)|$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Observe that $|s_1|$ is total.

We now state a number of propositions:

(9)[2] $(s_2 + s_3) + s_4 = s_2 + (s_3 + s_4)$.

(11)[3] $(s_2\,s_3)\,s_4 = s_2\,(s_3\,s_4)$.

(12) $(s_2 + s_3)\,s_4 = s_2\,s_4 + s_3\,s_4$.

(13) $s_4\,(s_2 + s_3) = s_4\,s_2 + s_4\,s_3$.

(14) $-s_1 = (-1_{\mathbb{C}})\,s_1$.

(15) $r\,(s_2\,s_3) = (r\,s_2)\,s_3$.

(16) $r\,(s_2\,s_3) = s_2\,(r\,s_3)$.

(17) $(s_2 - s_3)\,s_4 = s_2\,s_4 - s_3\,s_4$.

(18) $s_4\,s_2 - s_4\,s_3 = s_4\,(s_2 - s_3)$.

(19) $r\,(s_2 + s_3) = r\,s_2 + r\,s_3$.

(20) $(r \cdot p)\,s_1 = r\,(p\,s_1)$.

(21) $r\,(s_2 - s_3) = r\,s_2 - r\,s_3$.

(22) $r\,(s_2/s_1) = (r\,s_2)/s_1$.

(23) $s_2 - (s_3 + s_4) = s_2 - s_3 - s_4$.

(24) $1_{\mathbb{C}}\,s_1 = s_1$.

(25) $--s_1 = s_1$.

(26) $s_2 - -s_3 = s_2 + s_3$.

(27) $s_2 - (s_3 - s_4) = (s_2 - s_3) + s_4$.

(28) $s_2 + (s_3 - s_4) = (s_2 + s_3) - s_4$.

(29) $(-s_2)\,s_3 = -s_2\,s_3$ and $s_2\,{-s_3} = -s_2\,s_3$.

(30) If s_1 is non-zero, then ${s_1}^{-1}$ is non-zero.

(31) $({s_1}^{-1})^{-1} = s_1$.

(32) s_1 is non-zero and s_2 is non-zero iff $s_1\,s_2$ is non-zero.

(33) If s_1 is non-zero and s_2 is non-zero, then ${s_1}^{-1}\,{s_2}^{-1} = (s_1\,s_2)^{-1}$.

(34) If s_1 is non-zero, then $(s_2/s_1)\,s_1 = s_2$.

(35) If s_1 is non-zero and s_2 is non-zero, then $(s'_1/s_1)\,(s'_2/s_2) = (s'_1\,s'_2)/(s_1\,s_2)$.

(36) If s_1 is non-zero and s_2 is non-zero, then s_1/s_2 is non-zero.

[2] The proposition (8) has been removed.
[3] The proposition (10) has been removed.

(37) If s_1 is non-zero and s_2 is non-zero, then $(s_1/s_2)^{-1} = s_2/s_1$.

(38) $s_3\,(s_2/s_1) = (s_3\,s_2)/s_1$.

(39) If s_1 is non-zero and s_2 is non-zero, then $s_3/(s_1/s_2) = (s_3\,s_2)/s_1$.

(40) If s_1 is non-zero and s_2 is non-zero, then $s_3/s_1 = (s_3\,s_2)/(s_1\,s_2)$.

(41) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $r\,s_1$ is non-zero.

(42) If s_1 is non-zero, then $-s_1$ is non-zero.

(43) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $(r\,s_1)^{-1} = r^{-1}\,{s_1}^{-1}$.

(44) If s_1 is non-zero, then $(-s_1)^{-1} = (-1_{\mathbb{C}})\,{s_1}^{-1}$.

(45) If s_1 is non-zero, then $-s_2/s_1 = (-s_2)/s_1$ and $s_2/-s_1 = -s_2/s_1$.

(46) $s_2/s_1 + s'_2/s_1 = (s_2 + s'_2)/s_1$ and $s_2/s_1 - s'_2/s_1 = (s_2 - s'_2)/s_1$.

(47) If s_1 is non-zero and s'_1 is non-zero, then $s_2/s_1 + s'_2/s'_1 = (s_2\,s'_1 + s'_2\,s_1)/(s_1\,s'_1)$ and $s_2/s_1 - s'_2/s'_1 = (s_2\,s'_1 - s'_2\,s_1)/(s_1\,s'_1)$.

(48) If s_1 is non-zero and s'_1 is non-zero and s_2 is non-zero, then $s'_2/s_1/(s'_1/s_2) = (s'_2\,s_2)/(s_1\,s'_1)$.

(49) $|s_1\,s'_1| = |s_1|\,|s'_1|$.

(50) If s_1 is non-zero, then $|s_1|$ is non-zero.

(51) If s_1 is non-zero, then $|s_1|^{-1} = |{s_1}^{-1}|$.

(52) If s_1 is non-zero, then $|s'_1/s_1| = |s'_1|/|s_1|$.

(53) $|r\,s_1| = |r|\,|s_1|$.

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Received November 5, 1993

Published January 2, 2004