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Complex Spaces[1]

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Summary. We introduce the concept of *n*-dimensional complex space. We prove a number of simple but useful propositions concerning addition, nultiplication by scalars and similar basic concepts. We introduce metric and topology. We prove that *n*-dimensional complex space is a Hausdorff space and that it is regular.

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The articles [19], [7], [22], [1], [20], [15], [13], [21], [9], [4], [6], [5], [3], [16], [12], [2], [17], [18], [10], [8], [11], and [14] provide the notation and terminology for this paper.

We adopt the following rules: k, n are natural numbers, r, r', r_1 are real numbers, and c, c', c_1, c_2 are elements of $\mathbb{C}$.

In this article we present several logical schemes. The scheme *FuncDefUniq* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$ and a unary functor $\mathcal{F}$ yielding a set, and states that:

> Let f_1, f_2 be functions from $\mathcal{A}$ into $\mathcal{B}$. Suppose for every element x of $\mathcal{A}$ holds $f_1(x) = \mathcal{F}(x)$ and for every element x of $\mathcal{A}$ holds $f_2(x) = \mathcal{F}(x)$. Then $f_1 = f_2$

for all values of the parameters.

The scheme *BinOpDefuniq* deals with a non empty set $\mathcal{A}$ and a binary functor $\mathcal{F}$ yielding a set, and states that:

> Let o_1, o_2 be binary operations on $\mathcal{A}$. Suppose for all elements a, b of $\mathcal{A}$ holds $o_1(a, b) = \mathcal{F}(a,b)$ and for all elements a, b of $\mathcal{A}$ holds $o_2(a, b) = \mathcal{F}(a,b)$. Then $o_1 = o_2$

for all values of the parameters.

The binary operation $+_{\mathbb{C}}$ on $\mathbb{C}$ is defined as follows:

(Def. 1) For all c_1, c_2 holds $+_{\mathbb{C}}(c_1, c_2) = c_1 + c_2$.

Next we state several propositions:

(1) $+_{\mathbb{C}}$ is commutative.

(2) $+_{\mathbb{C}}$ is associative.

(3) $0_{\mathbb{C}}$ is a unity w.r.t. $+_{\mathbb{C}}$.

(4) $\mathbf{1}_{+_{\mathbb{C}}} = 0_{\mathbb{C}}$.

(5) $+_{\mathbb{C}}$ has a unity.

The unary operation $-_{\mathbb{C}}$ on $\mathbb{C}$ is defined by:

(Def. 2) For every c holds $-_{\mathbb{C}}(c) = -c$.

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One can prove the following three propositions:

(6) $-_{\mathbb{C}}$ is an inverse operation w.r.t. $+_{\mathbb{C}}$.

(7) $+_{\mathbb{C}}$ has an inverse operation.

(8) The inverse operation w.r.t. $+_{\mathbb{C}} = -_{\mathbb{C}}$.

The binary operation $-_{\mathbb{C}}$ on $\mathbb{C}$ is defined by:

(Def. 3) $-_{\mathbb{C}} = +_{\mathbb{C}} \circ (\mathrm{id}_{\mathbb{C}}, -_{\mathbb{C}})$.

The following proposition is true

(9) $-_{\mathbb{C}}(c_1, c_2) = c_1 - c_2$.

The binary operation $\cdot_{\mathbb{C}}$ on $\mathbb{C}$ is defined as follows:

(Def. 4) For all c_1, c_2 holds $\cdot_{\mathbb{C}}(c_1, c_2) = c_1 \cdot c_2$.

Next we state several propositions:

(10) $\cdot_{\mathbb{C}}$ is commutative.

(11) $\cdot_{\mathbb{C}}$ is associative.

(12) $1_{\mathbb{C}}$ is a unity w.r.t. $\cdot_{\mathbb{C}}$.

(13) $\mathbf{1}_{\cdot_{\mathbb{C}}} = 1_{\mathbb{C}}$.

(14) $\cdot_{\mathbb{C}}$ has a unity.

(15) $\cdot_{\mathbb{C}}$ is distributive w.r.t. $+_{\mathbb{C}}$.

Let us consider c. The functor $\cdot_{\mathbb{C}}^{c}$ yields a unary operation on $\mathbb{C}$ and is defined by:

(Def. 5) $\cdot_{\mathbb{C}}^{c} = (\cdot_{\mathbb{C}})^{\circ}(c, \mathrm{id}_{\mathbb{C}})$.

One can prove the following propositions:

(16) $\cdot_{\mathbb{C}}^{c}(c') = c \cdot c'$.

(17) $\cdot_{\mathbb{C}}^{c}$ is distributive w.r.t. $+_{\mathbb{C}}$.

The function $|\cdot|_{\mathbb{C}}$ from $\mathbb{C}$ into $\mathbb{R}$ is defined as follows:

(Def. 6) For every c holds $|\cdot|_{\mathbb{C}}(c) = |c|$.

In the sequel z, z_1, z_2 denote finite sequences of elements of $\mathbb{C}$.

Let us consider z_1, z_2. The functor $z_1 + z_2$ yields a finite sequence of elements of $\mathbb{C}$ and is defined by:

(Def. 7) $z_1 + z_2 = (+_{\mathbb{C}})^{\circ}(z_1, z_2)$.

The functor $z_1 - z_2$ yields a finite sequence of elements of $\mathbb{C}$ and is defined as follows:

(Def. 8) $z_1 - z_2 = (-_{\mathbb{C}})^{\circ}(z_1, z_2)$.

Let us consider z. The functor $-z$ yields a finite sequence of elements of $\mathbb{C}$ and is defined as follows:

(Def. 9) $-z = -_{\mathbb{C}} \cdot z$.

Let us consider c, z. The functor $c \cdot z$ yields a finite sequence of elements of $\mathbb{C}$ and is defined by:

(Def. 10) $c \cdot z = \cdot_{\mathbb{C}}^{c} \cdot z$.

Let us consider z. The functor $|z|$ yielding a finite sequence of elements of $\mathbb{R}$ is defined as follows:

(Def. 11) $|z| = |\cdot|_{\mathbb{C}} \cdot z$.

Let us consider n. The functor $\mathbb{C}^n$ yields a non empty set of finite sequences of $\mathbb{C}$ and is defined by:

(Def. 12) $\mathbb{C}^n = \mathbb{C}^n$.

Let us consider n. Note that $\mathbb{C}^n$ is non empty.

We adopt the following convention: x, z, z_1, z_2, z_3 are elements of $\mathbb{C}^n$ and A, B are subsets of $\mathbb{C}^n$.

Next we state several propositions:

(18) $\operatorname{len} z = n$.

(19) For every element z of $\mathbb{C}^0$ holds $z = \varepsilon_{\mathbb{C}}$.

(20) $\varepsilon_{\mathbb{C}}$ is an element of $\mathbb{C}^0$.

(21) If $k \in \operatorname{Seg} n$, then $z(k) \in \mathbb{C}$.

(23)[1] If for every k such that $k \in \operatorname{Seg} n$ holds $z_1(k) = z_2(k)$, then $z_1 = z_2$.

Let us consider n, z_1, z_2. Then $z_1 + z_2$ is an element of $\mathbb{C}^n$.

The following three propositions are true:

(24) If $k \in \operatorname{Seg} n$ and $c_1 = z_1(k)$ and $c_2 = z_2(k)$, then $(z_1 + z_2)(k) = c_1 + c_2$.

(25) $z_1 + z_2 = z_2 + z_1$.

(26) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$.

Let us consider n. The functor $0^n_{\mathbb{C}}$ yielding a finite sequence of elements of $\mathbb{C}$ is defined as follows:

(Def. 13) $0^n_{\mathbb{C}} = n \mapsto 0_{\mathbb{C}}$.

Let us consider n. Then $0^n_{\mathbb{C}}$ is an element of $\mathbb{C}^n$.

The following two propositions are true:

(27) If $k \in \operatorname{Seg} n$, then $0^n_{\mathbb{C}}(k) = 0_{\mathbb{C}}$.

(28) $z + 0^n_{\mathbb{C}} = z$ and $z = 0^n_{\mathbb{C}} + z$.

Let us consider n, z. Then $-z$ is an element of $\mathbb{C}^n$.

We now state several propositions:

(29) If $k \in \operatorname{Seg} n$ and $c = z(k)$, then $(-z)(k) = -c$.

(30) $z + -z = 0^n_{\mathbb{C}}$ and $-z + z = 0^n_{\mathbb{C}}$.

(31) If $z_1 + z_2 = 0^n_{\mathbb{C}}$, then $z_1 = -z_2$ and $z_2 = -z_1$.

(32) $--z = z$.

(33) If $-z_1 = -z_2$, then $z_1 = z_2$.

(34) If $z_1 + z = z_2 + z$ or $z_1 + z = z + z_2$, then $z_1 = z_2$.

(35) $-(z_1 + z_2) = -z_1 + -z_2$.

[1] The proposition (22) has been removed.

Let us consider n, z_1, z_2. Then $z_1 - z_2$ is an element of $\mathbb{C}^n$.
One can prove the following propositions:

(36) If $k \in \mathrm{Seg}\, n$ and $c_1 = z_1(k)$ and $c_2 = z_2(k)$, then $(z_1 - z_2)(k) = c_1 - c_2$.

(37) $z_1 - z_2 = z_1 + -z_2$.

(38) $z - 0^n_{\mathbb{C}} = z$.

(39) $0^n_{\mathbb{C}} - z = -z$.

(40) $z_1 - -z_2 = z_1 + z_2$.

(41) $-(z_1 - z_2) = z_2 - z_1$.

(42) $-(z_1 - z_2) = -z_1 + z_2$.

(43) $z - z = 0^n_{\mathbb{C}}$.

(44) If $z_1 - z_2 = 0^n_{\mathbb{C}}$, then $z_1 = z_2$.

(45) $z_1 - z_2 - z_3 = z_1 - (z_2 + z_3)$.

(46) $z_1 + (z_2 - z_3) = (z_1 + z_2) - z_3$.

(47) $z_1 - (z_2 - z_3) = (z_1 - z_2) + z_3$.

(48) $(z_1 - z_2) + z_3 = (z_1 + z_3) - z_2$.

(49) $z_1 = (z_1 + z) - z$.

(50) $z_1 + (z_2 - z_1) = z_2$.

(51) $z_1 = (z_1 - z) + z$.

Let us consider n, c, z. Then $c \cdot z$ is an element of $\mathbb{C}^n$.
The following propositions are true:

(52) If $k \in \mathrm{Seg}\, n$ and $c' = z(k)$, then $(c \cdot z)(k) = c \cdot c'$.

(53) $c_1 \cdot (c_2 \cdot z) = (c_1 \cdot c_2) \cdot z$.

(54) $(c_1 + c_2) \cdot z = c_1 \cdot z + c_2 \cdot z$.

(55) $c \cdot (z_1 + z_2) = c \cdot z_1 + c \cdot z_2$.

(56) $1_{\mathbb{C}} \cdot z = z$.

(57) $0_{\mathbb{C}} \cdot z = 0^n_{\mathbb{C}}$.

(58) $(-1_{\mathbb{C}}) \cdot z = -z$.

Let us consider n, z. Then $|z|$ is an element of $\mathbb{R}^n$.
The following four propositions are true:

(59) If $k \in \mathrm{Seg}\, n$ and $c = z(k)$, then $|z|(k) = |c|$.

(60) $|0^n_{\mathbb{C}}| = n \mapsto 0$.

(61) $|-z| = |z|$.

(62) $|c \cdot z| = |c| \cdot |z|$.

Let z be a finite sequence of elements of $\mathbb{C}$. The functor $|z|$ yields a real number and is defined by:

(Def. 14) $|z| = \sqrt{\sum^2 |z|}$.

Next we state a number of propositions:

(63) $|0^n_{\mathbb{C}}| = 0$.

(64) If $|z| = 0$, then $z = 0^n_{\mathbb{C}}$.

(65) $0 \leq |z|$.

(66) $|-z| = |z|$.

(67) $|c \cdot z| = |c| \cdot |z|$.

(68) $|z_1 + z_2| \leq |z_1| + |z_2|$.

(69) $|z_1 - z_2| \leq |z_1| + |z_2|$.

(70) $|z_1| - |z_2| \leq |z_1 + z_2|$.

(71) $|z_1| - |z_2| \leq |z_1 - z_2|$.

(72) $|z_1 - z_2| = 0$ iff $z_1 = z_2$.

(73) If $z_1 \neq z_2$, then $0 < |z_1 - z_2|$.

(74) $|z_1 - z_2| = |z_2 - z_1|$.

(75) $|z_1 - z_2| \leq |z_1 - z| + |z - z_2|$.

Let us consider n and let A be an element of $2^{\mathbb{C}^n}$. We say that A is open if and only if:

(Def. 15) For every x such that $x \in A$ there exists r such that $0 < r$ and for every z such that $|z| < r$ holds $x + z \in A$.

Let us consider n and let A be an element of $2^{\mathbb{C}^n}$. We say that A is closed if and only if:

(Def. 16) For every x such that for every r such that $r > 0$ there exists z such that $|z| < r$ and $x + z \in A$ holds $x \in A$.

Next we state four propositions:

(76) For every element A of $2^{\mathbb{C}^n}$ such that $A = \emptyset$ holds A is open.

(77) For every element A of $2^{\mathbb{C}^n}$ such that $A = \mathbb{C}^n$ holds A is open.

(78) Let A_1 be a family of subsets of $\mathbb{C}^n$. Suppose that for every element A of $2^{\mathbb{C}^n}$ such that $A \in A_1$ holds A is open. Let A be an element of $2^{\mathbb{C}^n}$. If $A = \bigcup A_1$, then A is open.

(79) Let A, B be subsets of $\mathbb{C}^n$. Suppose A is open and B is open. Let C be an element of $2^{\mathbb{C}^n}$. If $C = A \cap B$, then C is open.

Let us consider n, x, r. The functor $\mathrm{Ball}(x, r)$ yields a subset of $\mathbb{C}^n$ and is defined by:

(Def. 17) $\mathrm{Ball}(x, r) = \{z : |z - x| < r\}$.

The following three propositions are true:

(80) $z \in \mathrm{Ball}(x, r)$ iff $|x - z| < r$.

(81) If $0 < r$, then $x \in \mathrm{Ball}(x, r)$.

(82) $\mathrm{Ball}(z_1, r_1)$ is open.

Now we present two schemes. The scheme *SubsetFD* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and a unary predicate $\mathcal{P}$, and states that:

$\{\mathcal{F}(x); x$ ranges over elements of $\mathcal{A} : \mathcal{P}[x]\}$ is a subset of $\mathcal{B}$

for all values of the parameters.

The scheme *SubsetFD2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, and a binary predicate $\mathcal{P}$, and states that:

$\{\mathcal{F}(x,y); x$ ranges over elements of $\mathcal{A}, y$ ranges over elements of $\mathcal{B} : \mathcal{P}[x,y]\}$ is a subset of $\mathcal{C}$

for all values of the parameters.

Let us consider n, x, A. The functor $\rho(x,A)$ yields a real number and is defined as follows:

(Def. 18) For every subset X of $\mathbb{R}$ such that $X = \{|x-z| : z \in A\}$ holds $\rho(x,A) = \inf X$.

Let us consider n, A, r. The functor $\mathrm{Ball}(A,r)$ yielding a subset of $\mathbb{C}^n$ is defined by:

(Def. 19) $\mathrm{Ball}(A,r) = \{z : \rho(z,A) < r\}$.

We now state a number of propositions:

(83) If for every r' such that $r' > 0$ holds $r + r' > r_1$, then $r \geq r_1$.

(84) For every subset X of $\mathbb{R}$ and for every r such that $X \neq \emptyset$ and for every r' such that $r' \in X$ holds $r \leq r'$ holds $\inf X \geq r$.

(85) If $A \neq \emptyset$, then $\rho(x,A) \geq 0$.

(86) If $A \neq \emptyset$, then $\rho(x+z,A) \leq \rho(x,A) + |z|$.

(87) If $x \in A$, then $\rho(x,A) = 0$.

(88) If $x \notin A$ and $A \neq \emptyset$ and A is closed, then $\rho(x,A) > 0$.

(89) If $A \neq \emptyset$, then $|z_1 - x| + \rho(x,A) \geq \rho(z_1,A)$.

(90) $z \in \mathrm{Ball}(A,r)$ iff $\rho(z,A) < r$.

(91) If $0 < r$ and $x \in A$, then $x \in \mathrm{Ball}(A,r)$.

(92) If $0 < r$, then $A \subseteq \mathrm{Ball}(A,r)$.

(93) If $A \neq \emptyset$, then $\mathrm{Ball}(A,r_1)$ is open.

Let us consider n, A, B. The functor $\rho(A,B)$ yielding a real number is defined by:

(Def. 20) For every subset X of $\mathbb{R}$ such that $X = \{|x-z| : x \in A \land z \in B\}$ holds $\rho(A,B) = \inf X$.

Let X, Y be subsets of $\mathbb{R}$. The functor $X+Y$ yielding a subset of $\mathbb{R}$ is defined as follows:

(Def. 21) $X + Y = \{r + r_1 : r \in X \land r_1 \in Y\}$.

We now state several propositions:

(94) For all subsets X, Y of $\mathbb{R}$ such that $X \neq \emptyset$ and $Y \neq \emptyset$ holds $X + Y \neq \emptyset$.

(95) For all subsets X, Y of $\mathbb{R}$ such that X is lower bounded and Y is lower bounded holds $X+Y$ is lower bounded.

(96) For all subsets X, Y of $\mathbb{R}$ such that $X \neq \emptyset$ and X is lower bounded and $Y \neq \emptyset$ and Y is lower bounded holds $\inf(X+Y) = \inf X + \inf Y$.

(97) Let X, Y be subsets of $\mathbb{R}$. Suppose Y is lower bounded and $X \neq \emptyset$ and for every r such that $r \in X$ there exists r_1 such that $r_1 \in Y$ and $r_1 \leq r$. Then $\inf X \geq \inf Y$.

(98) If $A \neq \emptyset$ and $B \neq \emptyset$, then $\rho(A,B) \geq 0$.

(99) $\rho(A,B) = \rho(B,A)$.

(100) If $A \neq \emptyset$ and $B \neq \emptyset$, then $\rho(x,A) + \rho(x,B) \geq \rho(A,B)$.

(101) If A meets B, then $\rho(A,B) = 0$.

Let us consider n. The open subsets of $\mathbb{C}^n$ constitute a family of subsets of $\mathbb{C}^n$ defined by:

(Def. 22) The open subsets of $\mathbb{C}^n = \{A; A$ ranges over elements of $2^{\mathbb{C}^n}$: A is open$\}$.

The following proposition is true

(102) For every element A of $2^{\mathbb{C}^n}$ holds $A \in$ the open subsets of $\mathbb{C}^n$ iff A is open.

Let A be a non empty set and let t be a family of subsets of A. One can check that $\langle A, t \rangle$ is non empty.

Let us consider n. The n -dimensional complex space is a strict topological space and is defined by:

(Def. 23) The n-dimensional complex space $= \langle \mathbb{C}^n$, the open subsets of $\mathbb{C}^n \rangle$.

Let us consider n. Note that the n -dimensional complex space is non empty.

The following two propositions are true:

(103) The topology of (the n-dimensional complex space) = the open subsets of $\mathbb{C}^n$.

(104) The carrier of (the n-dimensional complex space) $= \mathbb{C}^n$.

In the sequel p is a point of the n-dimensional complex space and V is a subset of the n-dimensional complex space.

We now state several propositions:

(105) p is an element of $\mathbb{C}^n$.

(108)[2] For every subset A of $\mathbb{C}^n$ such that $A = V$ holds A is open iff V is open.

(109) For every subset A of $\mathbb{C}^n$ holds A is closed iff $A^{\rm c}$ is open.

(110) For every subset A of $\mathbb{C}^n$ such that $A = V$ holds A is closed iff V is closed.

(111) The n-dimensional complex space is a T_2 space.

(112) The n-dimensional complex space is a T_3 space.

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