JOURNAL OF FORMALIZED MATHEMATICS
Volume 14, Released 2002, Published 2003
Inst. of Computer Science, Univ. of Białystok

Preliminaries to Automatic Generation of Mizar Documentation for Circuits

Grzegorz Bancerek
Białystok Technical University

Adam Naumowicz
University of Białystok

Summary. In this paper we introduce technical notions used by a system which automatically generates Mizar documentation for specified circuits. They provide a ready for use elements needed to justify correctness of circuits' construction. We concentrate on the concept of stabilization and analyze one-gate circuits and their combinations.

MML Identifier: CIRCCMB3.

WWW: http://mizar.org/JFM/Vol14/circcmb3.html

The articles [21], [27], [25], [20], [11], [9], [28], [6], [12], [2], [3], [7], [1], [8], [14], [4], [10], [22], [26], [23], [5], [17], [18], [15], [16], [19], [13], and [24] provide the notation and terminology for this paper.

1. Stabilizing Circuits

One can prove the following proposition

(1) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and x be a set. If $x \in \mathrm{InputVertices}(S)$, then for every natural number n holds $(\mathrm{Following}(s,n))(x) = s(x)$.

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. We say that s is stabilizing if and only if:

(Def. 1) There exists a natural number n such that $\mathrm{Following}(s,n)$ is stable.

Let S be a non void circuit-like non empty many sorted signature and let A be a non-empty circuit of S. We say that A is stabilizing if and only if:

(Def. 2) Every state of A is stabilizing.

We say that A has a stabilization limit if and only if:

(Def. 3) There exists a natural number n such that for every state s of A holds $\mathrm{Following}(s,n)$ is stable.

Let S be a non void circuit-like non empty many sorted signature. Observe that every non-empty circuit of S which has a stabilization limit is also stabilizing.

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The functor $\mathrm{Result}(s)$ yields a state of A and is defined by:

(Def. 4) $\mathrm{Result}(s)$ is stable and there exists a natural number n such that $\mathrm{Result}(s) = \mathrm{Following}(s,n)$.

Let S be a non void circuit-like non empty many sorted signature, let A be a non-empty circuit of S, and let s be a state of A. Let us assume that s is stabilizing. The stabilization time of s is a natural number and is defined by the conditions (Def. 5).

(Def. 5)(i) Following(s, the stabilization time of s) is stable, and

(ii) for every natural number n such that $n <$ the stabilization time of s holds Following(s,n) is not stable.

We now state a number of propositions:

(2) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then Result(s) $=$ Following(s, the stabilization time of s).

(3) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s,n) is stable, then the stabilization time of $s \leq n$.

(4) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. If Following(s,n) is stable, then Result(s) $=$ Following(s,n).

(5) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, s be a state of A, and n be a natural number. Suppose s is stabilizing and $n \geq$ the stabilization time of s. Then Result(s) $=$ Following(s,n).

(6) Let S be a non void circuit-like non empty many sorted signature, A be a non-empty circuit of S, and s be a state of A. If s is stabilizing, then for every set x such that $x \in$ InputVertices(S) holds (Result(s))(x) $= s(x)$.

(7) Let S_1, S be non void circuit-like non empty many sorted signatures, A_1 be a non-empty circuit of S_1, A be a non-empty circuit of S, s be a state of A, and s_1 be a state of A_1. If $s_1 = s\restriction$the carrier of S_1, then for every vertex v_1 of S_1 holds $s_1(v_1) = s(v_1)$.

(8) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1{+\cdot}S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1{+\cdot}A_2$. Let s be a state of A, s_1 be a state of A_1, and s_2 be a state of A_2. Suppose $s_1 = s\restriction$the carrier of S_1 and $s_2 = s\restriction$the carrier of S_2 and s_1 is stabilizing and s_2 is stabilizing. Then s is stabilizing.

(9) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and InputVertices(S_2) misses InnerVertices(S_1). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1{+\cdot}S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1{+\cdot}A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 = s\restriction$the carrier of S_2 and s_2 is stabilizing. Then the stabilization time of $s =$ max(the stabilization time of s_1, the stabilization time of s_2).

(10) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1{+\cdot}S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1{+\cdot}A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 =$ Following(s, the stabilization time of s_1)$\restriction$the carrier of S_2 and s_2 is stabilizing. Then s is stabilizing.

(11) Let S_1, S_2 be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2). Let S be a non void circuit-like non empty many sorted signature. Suppose $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1 and A_2 be a non-empty circuit of S_2. Suppose $A_1 \approx A_2$. Let A be a non-empty circuit of S. Suppose $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 =$ Following$(s,$the stabilization time of $s_1)\restriction$the carrier of S_2 and s_2 is stabilizing. Then the stabilization time of $s =$ (the stabilization time of $s_1) +$ (the stabilization time of s_2).

(12) Let S_1, S_2, S be non void circuit-like non empty many sorted signatures. Suppose InputVertices(S_1) misses InnerVertices(S_2) and $S = S_1 {+\cdot} S_2$. Let A_1 be a non-empty circuit of S_1, A_2 be a non-empty circuit of S_2, and A be a non-empty circuit of S. Suppose $A_1 \approx A_2$ and $A = A_1 {+\cdot} A_2$. Let s be a state of A and s_1 be a state of A_1. Suppose $s_1 = s\restriction$the carrier of S_1 and s_1 is stabilizing. Let s_2 be a state of A_2. Suppose $s_2 =$ Following$(s,$the stabilization time of $s_1)\restriction$the carrier of S_2 and s_2 is stabilizing. Then Result$(s)\restriction$the carrier of $S_1 =$ Result(s_1).

2. One-gate Circuits

We now state three propositions:

(13) Let x be a set, X be a non empty finite set, n be a natural number, p be a finite sequence with length n, g be a function from X^n into X, and s be a state of 1GateCircuit(p,g). Then $s \cdot p$ is an element of X^n.

(14) For all sets x_1, x_2, x_3, x_4 holds rng$\langle x_1,x_2,x_3,x_4\rangle = \{x_1,x_2,x_3,x_4\}$.

(15) For all sets x_1, x_2, x_3, x_4, x_5 holds rng$\langle x_1,x_2,x_3,x_4,x_5\rangle = \{x_1,x_2,x_3,x_4,x_5\}$.

Let x_1, x_2, x_3, x_4 be sets. Then $\langle x_1,x_2,x_3,x_4\rangle$ is a finite sequence with length 4. Let x_5 be a set. Then $\langle x_1,x_2,x_3,x_4,x_5\rangle$ is a finite sequence with length 5.

Let S be a many sorted signature. We say that S is one-gate if and only if the condition (Def. 6) is satisfied.

(Def. 6) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that $S =$ 1GateCircStr(p, f).

Let S be a non empty many sorted signature and let A be an algebra over S. We say that A is one-gate if and only if the condition (Def. 7) is satisfied.

(Def. 7) There exists a non empty finite set X and there exists a natural number n and there exists a finite sequence p with length n and there exists a function f from X^n into X such that $S =$ 1GateCircStr(p, f) and $A =$ 1GateCircuit(p, f).

Let p be a finite sequence and let x be a set. One can verify that 1GateCircStr(p,x) is finite.

Let us note that every many sorted signature which is one-gate is also strict, non void, non empty, unsplit, and finite and has arity held in gates.

One can verify that every non empty many sorted signature which is one-gate has also denotation held in gates.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. Observe that 1GateCircStr(p, f) is one-gate.

One can verify that there exists a many sorted signature which is one-gate.

Let S be an one-gate many sorted signature. Observe that every circuit of S which is one-gate is also strict and non-empty.

Let X be a non empty finite set, let n be a natural number, let p be a finite sequence with length n, and let f be a function from X^n into X. One can check that 1GateCircuit(p, f) is one-gate.

Let S be an one-gate many sorted signature. Observe that there exists a circuit of S which is one-gate and non-empty.

Let S be an one-gate many sorted signature. The functor Output$S$ yields a vertex of S and is defined by:

(Def. 8) Output $S = \bigcup$(the operation symbols of S).

Let S be an one-gate many sorted signature. One can verify that Output S is pair.

We now state several propositions:

(16) Let S be an one-gate many sorted signature, p be a finite sequence, and x be a set. If $S = \mathrm{1GateCircStr}(p,x)$, then Output $S = \langle p, x\rangle$.

(17) For every one-gate many sorted signature S holds $\mathrm{InnerVertices}(S) = \{\mathrm{Output}\,S\}$.

(18) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, n be a natural number, X be a finite non empty set, f be a function from X^n into X, and p be a finite sequence with length n. If $A = \mathrm{1GateCircuit}(p, f)$, then $S = \mathrm{1GateCircStr}(p, f)$.

(19) Let n be a natural number, X be a finite non empty set, f be a function from X^n into X, p be a finite sequence with length n, and s be a state of $\mathrm{1GateCircuit}(p, f)$. Then $(\mathrm{Following}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(p, f)) = f(s \cdot p)$.

(20) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then $\mathrm{Following}(s)$ is stable.

Let S be a non void circuit-like non empty many sorted signature. Note that every non-empty circuit of S which is one-gate has also a stabilization limit.

One can prove the following propositions:

(21) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then $\mathrm{Result}(s) = \mathrm{Following}(s)$.

(22) Let S be an one-gate many sorted signature, A be an one-gate circuit of S, and s be a state of A. Then the stabilization time of $s \leq 1$.

In this article we present several logical schemes. The scheme *OneGate1Ex* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\,S) = \mathcal{F}(s(\mathcal{A}))$

for all values of the parameters.

The scheme *OneGate2Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, and a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A},\mathcal{B}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\,S) = \mathcal{F}(s(\mathcal{A}),s(\mathcal{B}))$

for all values of the parameters.

The scheme *OneGate3Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A},\mathcal{B},\mathcal{C}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\,S) = \mathcal{F}(s(\mathcal{A}),s(\mathcal{B}),s(\mathcal{C}))$

for all values of the parameters.

The scheme *OneGate4Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\,S) = \mathcal{F}(s(\mathcal{A}),s(\mathcal{B}),s(\mathcal{C}),s(\mathcal{D}))$

for all values of the parameters.

The scheme *OneGate5Ex* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, and a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, and states that:

> There exists an one-gate many sorted signature S and there exists an one-gate circuit A of S such that $\mathrm{InputVertices}(S) = \{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E}\}$ and for every state s of A holds $(\mathrm{Result}(s))(\mathrm{Output}\,S) = \mathcal{F}(s(\mathcal{A}),s(\mathcal{B}),s(\mathcal{C}),s(\mathcal{D}),s(\mathcal{E}))$

for all values of the parameters.

3. Mono-sorted Circuits

The following four propositions are true:

(23) For every constant function f holds $f = \mathrm{dom}\, f \longmapsto$ the value of f.

(24) For all non empty sets X, Y and for all natural numbers n, m such that $n \neq 0$ and $X^n = Y^m$ holds $X = Y$ and $n = m$.

(25) For all non empty many sorted signatures S_1, S_2 holds every vertex of S_1 is a vertex of $S_1 {+\cdot} S_2$.

(26) For all non empty many sorted signatures S_1, S_2 holds every vertex of S_2 is a vertex of $S_1 {+\cdot} S_2$.

Let X be a non empty finite set. A non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates is said to be a signature over X if it satisfies the condition (Def. 9).

(Def. 9) There exists a circuit A of it such that the sorts of A are constant and the value of the sorts of $A = X$ and A has denotation held in gates.

The following proposition is true

(27) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then $\mathrm{1GateCircStr}(p, f)$ is a signature over X.

Let X be a non empty finite set. Note that there exists a signature over X which is strict and one-gate.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then $\mathrm{1GateCircStr}(p, f)$ is a strict signature over X.

Let X be a non empty finite set and let S be a signature over X. A circuit of S is called a circuit over X and S if:

(Def. 10) It has denotation held in gates and the sorts of it are constant and the value of the sorts of it $= X$.

Let X be a non empty finite set and let S be a signature over X. Note that every circuit over X and S is non-empty and has denotation held in gates.

Next we state the proposition

(28) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, and p be a finite sequence with length n. Then $\mathrm{1GateCircuit}(p, f)$ is a circuit over X and $\mathrm{1GateCircStr}(p, f)$.

Let X be a non empty finite set and let S be an one-gate signature over X. One can verify that there exists a circuit over X and S which is strict and one-gate.

Let X be a non empty finite set and let S be a signature over X. Note that there exists a circuit over X and S which is strict.

Let n be a natural number, let X be a non empty finite set, let f be a function from X^n into X, and let p be a finite sequence with length n. Then $\mathrm{1GateCircuit}(p, f)$ is a strict circuit over X and $\mathrm{1GateCircStr}(p, f)$.

We now state four propositions:

(30)[1] Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 \approx A_2$.

(31) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 {+\cdot} A_2$ is a circuit of $S_1 {+\cdot} S_2$.

[1] The proposition (29) has been removed.

(32) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then $A_1 {+\cdot} A_2$ has denotation held in gates.

(33) Let X be a non empty finite set, S_1, S_2 be signatures over X, A_1 be a circuit over X and S_1, and A_2 be a circuit over X and S_2. Then the sorts of $A_1 {+\cdot} A_2$ are constant and the value of the sorts of $A_1 {+\cdot} A_2 = X$.

Let S_1, S_2 be finite non empty many sorted signatures. Observe that $S_1 {+\cdot} S_2$ is finite.

Let X be a non empty finite set and let S_1, S_2 be signatures over X. One can verify that $S_1 {+\cdot} S_2$ has denotation held in gates.

Let X be a non empty finite set and let S_1, S_2 be signatures over X. Then $S_1 {+\cdot} S_2$ is a strict signature over X.

Let X be a non empty finite set, let S_1, S_2 be signatures over X, let A_1 be a circuit over X and S_1, and let A_2 be a circuit over X and S_2. Then $A_1 {+\cdot} A_2$ is a strict circuit over X and $S_1 {+\cdot} S_2$.

The following propositions are true:

(34) For all sets x, y holds $\mathrm{rk}(x) \in \mathrm{rk}(\langle x, y \rangle)$ and $\mathrm{rk}(y) \in \mathrm{rk}(\langle x, y \rangle)$.

(35) Let S be a finite non void non empty unsplit many sorted signature with arity held in gates with denotation held in gates and A be a non-empty circuit of S such that A has denotation held in gates. Then A has a stabilization limit.

Let X be a non empty finite set and let S be a finite signature over X. Note that every circuit over X and S has a stabilization limit.

Now we present three schemes. The scheme *1AryDef* deals with a non empty set $\mathcal{A}$ and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^1$ into $\mathcal{A}$ such that for every element x of $\mathcal{A}$ holds $f(\langle x \rangle) = \mathcal{F}(x)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^1$ into $\mathcal{A}$ such that for every element x of $\mathcal{A}$ holds $f_1(\langle x \rangle) = \mathcal{F}(x)$ and for every element x of $\mathcal{A}$ holds $f_2(\langle x \rangle) = \mathcal{F}(x)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme *2AryDef* deals with a non empty set $\mathcal{A}$ and a binary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^2$ into $\mathcal{A}$ such that for all elements x, y of $\mathcal{A}$ holds $f(\langle x,y \rangle) = \mathcal{F}(x,y)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^2$ into $\mathcal{A}$ such that for all elements x, y of $\mathcal{A}$ holds $f_1(\langle x,y \rangle) = \mathcal{F}(x,y)$ and for all elements x, y of $\mathcal{A}$ holds $f_2(\langle x,y \rangle) = \mathcal{F}(x,y)$ holds $f_1 = f_2$

for all values of the parameters.

The scheme *3AryDef* deals with a non empty set $\mathcal{A}$ and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathcal{A}^3$ into $\mathcal{A}$ such that for all elements x, y, z of $\mathcal{A}$ holds $f(\langle x,y,z \rangle) = \mathcal{F}(x,y,z)$, and

(ii) for all functions f_1, f_2 from $\mathcal{A}^3$ into $\mathcal{A}$ such that for all elements x, y, z of $\mathcal{A}$ holds $f_1(\langle x,y,z \rangle) = \mathcal{F}(x,y,z)$ and for all elements x, y, z of $\mathcal{A}$ holds $f_2(\langle x,y,z \rangle) = \mathcal{F}(x,y,z)$ holds $f_1 = f_2$

for all values of the parameters.

One can prove the following three propositions:

(36) For every function f and for every set x such that $x \in \mathrm{dom}\, f$ holds $f \cdot \langle x \rangle = \langle f(x) \rangle$.

(37) Let f be a function and x_1, x_2, x_3, x_4 be sets. If $x_1 \in \mathrm{dom}\, f$ and $x_2 \in \mathrm{dom}\, f$ and $x_3 \in \mathrm{dom}\, f$ and $x_4 \in \mathrm{dom}\, f$, then $f \cdot \langle x_1,x_2,x_3,x_4 \rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4) \rangle$.

(38) Let f be a function and x_1, x_2, x_3, x_4, x_5 be sets. Suppose $x_1 \in \mathrm{dom}\, f$ and $x_2 \in \mathrm{dom}\, f$ and $x_3 \in \mathrm{dom}\, f$ and $x_4 \in \mathrm{dom}\, f$ and $x_5 \in \mathrm{dom}\, f$. Then $f \cdot \langle x_1,x_2,x_3,x_4,x_5 \rangle = \langle f(x_1), f(x_2), f(x_3), f(x_4), f(x_5) \rangle$.

Now we present several schemes. The scheme *OneGate1Result* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and a function $\mathcal{C}$ from $\mathcal{B}^1$ into $\mathcal{B}$, and states that:

For every state s of 1GateCircuit$(\langle\mathcal{A}\rangle,\mathcal{C})$ and for every element a_1 of $\mathcal{B}$ such that $a_1 = s(\mathcal{A})$ holds $(\mathrm{Result}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(\langle\mathcal{A}\rangle,\mathcal{C})) = \mathcal{F}(a_1)$

provided the parameters meet the following condition:

- For every function g from $\mathcal{B}^1$ into $\mathcal{B}$ holds $g = \mathcal{C}$ iff for every element a_1 of $\mathcal{B}$ holds $g(\langle a_1\rangle) = \mathcal{F}(a_1)$.

The scheme *OneGate2Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, and a function $\mathcal{D}$ from $\mathcal{C}^2$ into $\mathcal{C}$, and states that:

For every state s of 1GateCircuit$(\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{D})$ and for all elements a_1, a_2 of $\mathcal{C}$ such that $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ holds $(\mathrm{Result}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B}\rangle,\mathcal{D})) = \mathcal{F}(a_1,a_2)$

provided the parameters meet the following requirement:

- For every function g from $\mathcal{C}^2$ into $\mathcal{C}$ holds $g = \mathcal{D}$ iff for all elements a_1, a_2 of $\mathcal{C}$ holds $g(\langle a_1,a_2\rangle) = \mathcal{F}(a_1,a_2)$.

The scheme *OneGate3Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and a function $\mathcal{E}$ from $\mathcal{D}^3$ into $\mathcal{D}$, and states that:

Let s be a state of 1GateCircuit$(\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{E})$ and a_1, a_2, a_3 be elements of $\mathcal{D}$. If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$, then $(\mathrm{Result}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B},\mathcal{C}\rangle,\mathcal{E})) = \mathcal{F}(a_1,a_2,a_3)$

provided the following requirement is met:

- For every function g from $\mathcal{D}^3$ into $\mathcal{D}$ holds $g = \mathcal{E}$ iff for all elements a_1, a_2, a_3 of $\mathcal{D}$ holds $g(\langle a_1,a_2,a_3\rangle) = \mathcal{F}(a_1,a_2,a_3)$.

The scheme *OneGate4Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, and a function $\mathcal{F}$ from $\mathcal{E}^4$ into $\mathcal{E}$, and states that:

Let s be a state of 1GateCircuit$(\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\rangle,\mathcal{F})$ and a_1, a_2, a_3, a_4 be elements of $\mathcal{E}$. If $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$, then $(\mathrm{Result}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\rangle,\mathcal{F})) = \mathcal{F}(a_1,a_2,a_3,a_4)$

provided the following condition is met:

- Let g be a function from $\mathcal{E}^4$ into $\mathcal{E}$. Then $g = \mathcal{F}$ if and only if for all elements a_1, a_2, a_3, a_4 of $\mathcal{E}$ holds $g(\langle a_1,a_2,a_3,a_4\rangle) = \mathcal{F}(a_1,a_2,a_3,a_4)$.

The scheme *OneGate5Result* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, and a function $\mathcal{G}$ from $\mathcal{F}^5$ into $\mathcal{F}$, and states that:

Let s be a state of 1GateCircuit$(\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E}\rangle,\mathcal{G})$ and a_1, a_2, a_3, a_4, a_5 be elements of $\mathcal{F}$. Suppose $a_1 = s(\mathcal{A})$ and $a_2 = s(\mathcal{B})$ and $a_3 = s(\mathcal{C})$ and $a_4 = s(\mathcal{D})$ and $a_5 = s(\mathcal{E})$. Then $(\mathrm{Result}(s))(\mathrm{Output}\,\mathrm{1GateCircStr}(\langle\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E}\rangle,\mathcal{G})) = \mathcal{F}(a_1,a_2,a_3,a_4,a_5)$

provided the following condition is met:

- Let g be a function from $\mathcal{F}^5$ into $\mathcal{F}$. Then $g = \mathcal{G}$ if and only if for all elements a_1, a_2, a_3, a_4, a_5 of $\mathcal{F}$ holds $g(\langle a_1,a_2,a_3,a_4,a_5\rangle) = \mathcal{F}(a_1,a_2,a_3,a_4,a_5)$.

4. Input of a Compound Circuit

The following propositions are true:

(39) Let n be a natural number, X be a non empty finite set, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. If $\mathrm{rng}\,p \subseteq$ the carrier of S and $\mathrm{Output}\,\mathrm{1GateCircStr}(p,f) \notin \mathrm{InputVertices}(S)$, then $\mathrm{InputVertices}(S{+\cdot}\,\mathrm{1GateCircStr}(p,f)) = \mathrm{InputVertices}(S)$.

(40) Let X_1, X_2 be sets, X be a non empty finite set, n be a natural number, f be a function from X^n into X, p be a finite sequence with length n, and S be a signature over X. Suppose $\mathrm{rng}\,p = X_1 \cup X_2$ and $X_1 \subseteq$ the carrier of S and X_2 misses InnerVertices(S) and $\mathrm{Output}\,\mathrm{1GateCircStr}(p,f) \notin \mathrm{InputVertices}(S)$. Then $\mathrm{InputVertices}(S{+\cdot}\,\mathrm{1GateCircStr}(p,f)) = \mathrm{InputVertices}(S) \cup X_2$.

(41) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X. If $x_1 \in$ the carrier of S and Output 1GateCircStr($\langle x_1 \rangle, f$) $\notin$ InputVertices(S), then InputVertices(S+· 1GateCircStr($\langle x_1 \rangle, f$)) = InputVertices(S).

(42) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2 \rangle, f$)) = InputVertices($S$) $\cup \{x_2\}$.

(43) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2 \rangle, f$)) = InputVertices($S$) $\cup \{x_1\}$.

(44) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2 \rangle, f$)) = InputVertices($S$).

(45) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_2, x_3\}$.

(46) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_1, x_3\}$.

(47) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_3 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_1, x_2\}$.

(48) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_3\}$.

(49) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_3 \in$ the carrier of S and $x_2 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_2\}$.

(50) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and $x_1 \notin$ InnerVertices(S) and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$) $\cup \{x_1\}$.

(51) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X. Suppose $x_1 \in$ the carrier of S and $x_2 \in$ the carrier of S and $x_3 \in$ the carrier of S and Output 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) $\notin$ InputVertices(S). Then InputVertices(S+· 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$)) = InputVertices($S$).

5. RESULT OF A COMPOUND CIRCUIT

We now state the proposition

(52) Let X be a non empty finite set, S be a finite signature over X, A be a circuit over X and S, n be a natural number, f be a function from X^n into X, and p be a finite sequence with length n. Suppose Output1GateCircStr$(p, f) \notin$ InputVertices(S). Let s be a state of $A +\cdot$ 1GateCircuit(p, f) and s' be a state of A. Suppose $s' = s \restriction$the carrier of S. Then the stabilization time of $s \leq 1 +$ the stabilization time of s'.

Now we present several schemes. The scheme *Comb1CircResult* deals with a set $\mathcal{A}$, a non empty finite set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a finite signature $\mathcal{C}$ over $\mathcal{B}$, a circuit $\mathcal{D}$ over $\mathcal{B}$ and $\mathcal{C}$, and a function $\mathcal{E}$ from $\mathcal{B}^1$ into $\mathcal{B}$, and states that:

Let s be a state of $\mathcal{D} +\cdot$ 1GateCircuit$(\langle \mathcal{A} \rangle, \mathcal{E})$ and s' be a state of $\mathcal{D}$. Suppose $s' = s \restriction$the carrier of $\mathcal{C}$. Let a_1 be an element of $\mathcal{B}$. Suppose if $\mathcal{A} \in$ InnerVertices$(\mathcal{C})$, then $a_1 = ($Result$(s'))(\mathcal{A})$ and if $\mathcal{A} \notin$ InnerVertices$(\mathcal{C})$, then $a_1 = s(\mathcal{A})$. Then $($Result$(s))($Output1GateCircStr$(\langle \mathcal{A} \rangle, \mathcal{E})) = \mathcal{F}(a_1)$

provided the parameters meet the following conditions:

- For every function g from $\mathcal{B}^1$ into $\mathcal{B}$ holds $g = \mathcal{E}$ iff for every element a_1 of $\mathcal{B}$ holds $g(\langle a_1 \rangle) = \mathcal{F}(a_1)$, and
- Output1GateCircStr$(\langle \mathcal{A} \rangle, \mathcal{E}) \notin$ InputVertices$(\mathcal{C})$.

The scheme *Comb2CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, a non empty finite set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a finite signature $\mathcal{D}$ over $\mathcal{C}$, a circuit $\mathcal{E}$ over $\mathcal{C}$ and $\mathcal{D}$, and a function $\mathcal{F}$ from $\mathcal{C}^2$ into $\mathcal{C}$, and states that:

Let s be a state of $\mathcal{E} +\cdot$ 1GateCircuit$(\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F})$ and s' be a state of $\mathcal{E}$. Suppose $s' = s \restriction$the carrier of $\mathcal{D}$. Let a_1, a_2 be elements of $\mathcal{C}$. Suppose if $\mathcal{A} \in$ InnerVertices$(\mathcal{D})$, then $a_1 = ($Result$(s'))(\mathcal{A})$ and if $\mathcal{A} \notin$ InnerVertices$(\mathcal{D})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices$(\mathcal{D})$, then $a_2 = ($Result$(s'))(\mathcal{B})$ and if $\mathcal{B} \notin$ InnerVertices$(\mathcal{D})$, then $a_2 = s(\mathcal{B})$. Then $($Result$(s))($Output1GateCircStr$(\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F})) = \mathcal{F}(a_1, a_2)$

provided the following conditions are satisfied:

- For every function g from $\mathcal{C}^2$ into $\mathcal{C}$ holds $g = \mathcal{F}$ iff for all elements a_1, a_2 of $\mathcal{C}$ holds $g(\langle a_1, a_2 \rangle) = \mathcal{F}(a_1, a_2)$, and
- Output1GateCircStr$(\langle \mathcal{A}, \mathcal{B} \rangle, \mathcal{F}) \notin$ InputVertices$(\mathcal{D})$.

The scheme *Comb3CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a non empty finite set $\mathcal{D}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, a finite signature $\mathcal{E}$ over $\mathcal{D}$, a circuit $\mathcal{F}$ over $\mathcal{D}$ and $\mathcal{E}$, and a function $\mathcal{G}$ from $\mathcal{D}^3$ into $\mathcal{D}$, and states that:

Let s be a state of $\mathcal{F} +\cdot$ 1GateCircuit$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G})$ and s' be a state of $\mathcal{F}$. Suppose $s' = s \restriction$the carrier of $\mathcal{E}$. Let a_1, a_2, a_3 be elements of $\mathcal{D}$. Suppose that

(i) if $\mathcal{A} \in$ InnerVertices$(\mathcal{E})$, then $a_1 = ($Result$(s'))(\mathcal{A})$,
(ii) if $\mathcal{A} \notin$ InnerVertices$(\mathcal{E})$, then $a_1 = s(\mathcal{A})$,
(iii) if $\mathcal{B} \in$ InnerVertices$(\mathcal{E})$, then $a_2 = ($Result$(s'))(\mathcal{B})$,
(iv) if $\mathcal{B} \notin$ InnerVertices$(\mathcal{E})$, then $a_2 = s(\mathcal{B})$,
(v) if $\mathcal{C} \in$ InnerVertices$(\mathcal{E})$, then $a_3 = ($Result$(s'))(\mathcal{C})$, and
(vi) if $\mathcal{C} \notin$ InnerVertices$(\mathcal{E})$, then $a_3 = s(\mathcal{C})$.

Then $($Result$(s))($Output1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G})) = \mathcal{F}(a_1, a_2, a_3)$

provided the following conditions are met:

- For every function g from $\mathcal{D}^3$ into $\mathcal{D}$ holds $g = \mathcal{G}$ iff for all elements a_1, a_2, a_3 of $\mathcal{D}$ holds $g(\langle a_1, a_2, a_3 \rangle) = \mathcal{F}(a_1, a_2, a_3)$, and
- Output1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C} \rangle, \mathcal{G}) \notin$ InputVertices$(\mathcal{E})$.

The scheme *Comb4CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, a non empty finite set $\mathcal{E}$, a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, a finite signature $\mathcal{F}$ over $\mathcal{E}$, a circuit $\mathcal{G}$ over $\mathcal{E}$ and $\mathcal{F}$, and a function $\mathcal{H}$ from $\mathcal{E}^4$ into $\mathcal{E}$, and states that:

Let s be a state of $\mathcal{G} +\cdot$ 1GateCircuit$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H})$ and s' be a state of $\mathcal{G}$. Suppose $s' = s \restriction$the carrier of $\mathcal{F}$. Let a_1, a_2, a_3, a_4 be elements of $\mathcal{E}$. Suppose that if $\mathcal{A} \in$ InnerVertices$(\mathcal{F})$, then $a_1 = ($Result$(s'))(\mathcal{A})$ and if $\mathcal{A} \notin$ InnerVertices$(\mathcal{F})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices$(\mathcal{F})$, then $a_2 = ($Result$(s'))(\mathcal{B})$ and if $\mathcal{B} \notin$ InnerVertices$(\mathcal{F})$, then $a_2 = s(\mathcal{B})$ and if $\mathcal{C} \in$ InnerVertices$(\mathcal{F})$, then $a_3 = ($Result$(s'))(\mathcal{C})$ and if $\mathcal{C} \notin$ InnerVertices$(\mathcal{F})$, then $a_3 = s(\mathcal{C})$ and if $\mathcal{D} \in$ InnerVertices$(\mathcal{F})$, then $a_4 = ($Result$(s'))(\mathcal{D})$ and if $\mathcal{D} \notin$ InnerVertices$(\mathcal{F})$, then $a_4 = s(\mathcal{D})$. Then $($Result$(s))($Output1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H})) = \mathcal{F}(a_1, a_2, a_3, a_4)$

provided the parameters meet the following conditions:

- Let g be a function from $\mathcal{E}^4$ into $\mathcal{E}$. Then $g = \mathcal{H}$ if and only if for all elements a_1, a_2, a_3, a_4 of $\mathcal{E}$ holds $g(\langle a_1, a_2, a_3, a_4 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4)$, and
- Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \rangle, \mathcal{H}) \notin$ InputVertices$(\mathcal{F})$.

The scheme *Comb5CircResult* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, a non empty finite set $\mathcal{F}$, a 5-ary functor $\mathcal{F}$ yielding an element of $\mathcal{F}$, a finite signature $\mathcal{G}$ over $\mathcal{F}$, a circuit $\mathcal{H}$ over $\mathcal{F}$ and $\mathcal{G}$, and a function I from $\mathcal{F}^5$ into $\mathcal{F}$, and states that:

> Let s be a state of $\mathcal{H}$+· 1GateCircuit$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, I)$ and s' be a state of $\mathcal{H}$. Suppose $s' = s \restriction$the carrier of $\mathcal{G}$. Let a_1, a_2, a_3, a_4, a_5 be elements of $\mathcal{F}$. Suppose that if $\mathcal{A} \in$ InnerVertices$(\mathcal{G})$, then $a_1 = (\text{Result}(s'))(\mathcal{A})$ and if $\mathcal{A} \notin$ InnerVertices$(\mathcal{G})$, then $a_1 = s(\mathcal{A})$ and if $\mathcal{B} \in$ InnerVertices$(\mathcal{G})$, then $a_2 = (\text{Result}(s'))(\mathcal{B})$ and if $\mathcal{B} \notin$ InnerVertices$(\mathcal{G})$, then $a_2 = s(\mathcal{B})$ and if $\mathcal{C} \in$ InnerVertices$(\mathcal{G})$, then $a_3 = (\text{Result}(s'))(\mathcal{C})$ and if $\mathcal{C} \notin$ InnerVertices$(\mathcal{G})$, then $a_3 = s(\mathcal{C})$ and if $\mathcal{D} \in$ InnerVertices$(\mathcal{G})$, then $a_4 = (\text{Result}(s'))(\mathcal{D})$ and if $\mathcal{D} \notin$ InnerVertices$(\mathcal{G})$, then $a_4 = s(\mathcal{D})$ and if $\mathcal{E} \in$ InnerVertices$(\mathcal{G})$, then $a_5 = (\text{Result}(s'))(\mathcal{E})$ and if $\mathcal{E} \notin$ InnerVertices$(\mathcal{G})$, then $a_5 = s(\mathcal{E})$. Then $(\text{Result}(s))$(Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, I)) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$

provided the following conditions are met:

- Let g be a function from $\mathcal{F}^5$ into $\mathcal{F}$. Then $g = I$ if and only if for all elements a_1, a_2, a_3, a_4, a_5 of $\mathcal{F}$ holds $g(\langle a_1, a_2, a_3, a_4, a_5 \rangle) = \mathcal{F}(a_1, a_2, a_3, a_4, a_5)$, and
- Output 1GateCircStr$(\langle \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E} \rangle, I) \notin$ InputVertices$(\mathcal{G})$.

6. Inputs Without Pairs

Let S be a non empty many sorted signature. We say that S has nonpair inputs if and only if:

(Def. 11) InputVertices(S) has no pairs.

One can check that $\mathbb{N}$ has no pairs. Let X be a set with no pairs. One can verify that every subset of X has no pairs.

Let us observe that every function which is natural-yielding is also nonpair yielding.

Let us observe that every finite sequence of elements of $\mathbb{N}$ is natural-yielding.

One can verify that there exists a finite sequence which is one-to-one and natural-yielding.

Let n be a natural number. Note that there exists a finite sequence with length n which is one-to-one and natural-yielding.

Let p be a nonpair yielding finite sequence and let f be a set. Observe that 1GateCircStr(p, f) has nonpair inputs.

One can verify that there exists an one-gate many sorted signature which has nonpair inputs. Let X be a non empty finite set. One can check that there exists an one-gate signature over X which has nonpair inputs.

Let S be a non empty many sorted signature with nonpair inputs. Note that InputVertices(S) has no pairs.

Next we state the proposition

(53) Let S be a non empty many sorted signature with nonpair inputs and x be a vertex of S. If x is pair, then $x \in$ InnerVertices(S).

Let S be an unsplit non empty many sorted signature with arity held in gates. Observe that InnerVertices(S) is relation-like.

Let S be an unsplit non empty non void many sorted signature with denotation held in gates. One can check that InnerVertices(S) is relation-like.

Let S_1, S_2 be unsplit non empty many sorted signatures with arity held in gates with nonpair inputs. Note that S_1+·S_2 has nonpair inputs.

Next we state two propositions:

(54) For every non pair set x and for every binary relation R holds $x \notin R$.

(55) Let x_1 be a set, X be a non empty finite set, f be a function from X^1 into X, and S be a signature over X with nonpair inputs. If $x_1 \in$ the carrier of S or x_1 is non pair, then $S +\cdot$ 1GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, and let f be a function from X^1 into X. One can verify that $S +\cdot$ 1GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a non pair set, and let f be a function from X^1 into X. Note that $S +\cdot$ 1GateCircStr($\langle x_1 \rangle, f$) has nonpair inputs.

We now state the proposition

(56) Let x_1, x_2 be sets, X be a non empty finite set, f be a function from X^2 into X, and S be a signature over X with nonpair inputs. Suppose $x_1 \in$ the carrier of S or x_1 is non pair but $x_2 \in$ the carrier of S or x_2 is non pair. Then $S +\cdot$ 1GateCircStr($\langle x_1, x_2 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1 be a vertex of S, let n_2 be a non pair set, and let f be a function from X^2 into X. One can check that $S +\cdot$ 1GateCircStr($\langle x_1, n_2 \rangle, f$) has nonpair inputs and $S +\cdot$ 1GateCircStr($\langle n_2, x_1 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2 be vertices of S, and let f be a function from X^2 into X. One can verify that $S +\cdot$ 1GateCircStr($\langle x_1, x_2 \rangle, f$) has nonpair inputs.

We now state the proposition

(57) Let x_1, x_2, x_3 be sets, X be a non empty finite set, f be a function from X^3 into X, and S be a signature over X with nonpair inputs. Suppose that

(i) $x_1 \in$ the carrier of S or x_1 is non pair,

(ii) $x_2 \in$ the carrier of S or x_2 is non pair, and

(iii) $x_3 \in$ the carrier of S or x_3 is non pair.

Then $S +\cdot$ 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2 be vertices of S, let n be a non pair set, and let f be a function from X^3 into X. One can verify the following observations:

* $S +\cdot$ 1GateCircStr($\langle x_1, x_2, n \rangle, f$) has nonpair inputs,
* $S +\cdot$ 1GateCircStr($\langle x_1, n, x_2 \rangle, f$) has nonpair inputs, and
* $S +\cdot$ 1GateCircStr($\langle n, x_1, x_2 \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x be a vertex of S, let n_1, n_2 be non pair sets, and let f be a function from X^3 into X. One can check the following observations:

* $S +\cdot$ 1GateCircStr($\langle x, n_1, n_2 \rangle, f$) has nonpair inputs,
* $S +\cdot$ 1GateCircStr($\langle n_1, x, n_2 \rangle, f$) has nonpair inputs, and
* $S +\cdot$ 1GateCircStr($\langle n_1, n_2, x \rangle, f$) has nonpair inputs.

Let X be a non empty finite set, let S be a signature over X with nonpair inputs, let x_1, x_2, x_3 be vertices of S, and let f be a function from X^3 into X. One can check that $S +\cdot$ 1GateCircStr($\langle x_1, x_2, x_3 \rangle, f$) has nonpair inputs.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Grzegorz Bancerek. Sequences of ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal2.html.

[3] Grzegorz Bancerek. Tarski's classes and ranks. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/classes1.html.

[4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[5] Grzegorz Bancerek and Yatsuka Nakamura. Full adder circuit. Part I. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/facirc_1.html.

[6] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[7] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[8] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[9] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[10] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseq_2.html.

[11] Jing-Chao Chen. A small computer model with push-down stack. *Journal of Formalized Mathematics*, 11, 1999. http://mizar.org/JFM/Vol11/scmpds_1.html.

[12] Agata Darmochwał. Finite sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finset_1.html.

[13] Jarosław Kotowicz. Monotone real sequences. Subsequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/seqm_3.html.

[14] Jarosław Kotowicz. The limit of a real function at infinity. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/limfunc1.html.

[15] Yatsuka Nakamura and Grzegorz Bancerek. Combining of circuits. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/circcomb.html.

[16] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, I. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/pre_circ.html.

[17] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, II. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/msafree2.html.

[18] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Introduction to circuits, II. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/circuit2.html.

[19] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/binarith.html.

[20] Andrzej Trybulec. Enumerated sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/enumset1.html.

[21] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[22] Andrzej Trybulec. Many-sorted sets. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/pboole.html.

[23] Andrzej Trybulec. Many sorted algebras. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/msualg_1.html.

[24] Andrzej Trybulec. Moore-Smith convergence. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/yellow_6.html.

[25] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. http://mizar.org/JFM/Addenda/numbers.html.

[26] Wojciech A. Trybulec. Groups. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/group_1.html.

[27] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[28] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

Received July 26, 2002

Published January 2, 2004