JOURNAL OF FORMALIZED MATHEMATICS
Volume 6, Released 1994, Published 2003
Inst. of Computer Science, Univ. of Białystok

Categorial Categories and Slice Categories

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Summary. By categorial categories we mean categories with categories as objects and morphisms of the form (C_1, C_2, F), where C_1 and C_2 are categories and F is a functor from C_1 into C_2.

MML Identifier: `CAT_5`.

WWW: `http://mizar.org/JFM/Vol6/cat_5.html`

The articles [10], [6], [13], [11], [9], [14], [2], [3], [7], [12], [5], [4], [8], and [1] provide the notation and terminology for this paper.

1. Categories with Triple-like Morphisms

Let D_1, D_2, D be non empty sets and let x be an element of $[:[:D_1, D_2:], D:]$. Then $x_{\mathbf{1},\mathbf{1}}$ is an element of D_1. Then $x_{\mathbf{1},\mathbf{2}}$ is an element of D_2.

Let D_1, D_2 be non empty sets and let x be an element of $[:D_1, D_2:]$. Then $x_{\mathbf{2}}$ is an element of D_2.

The following proposition is true

(1) Let C, D be category structures. Suppose the category structure of C = the category structure of D. If C is category-like, then D is category-like.

Let I_1 be a category structure. We say that I_1 has triple-like morphisms if and only if:

(Def. 1) For every morphism f of I_1 there exists a set x such that $f = \langle\langle \mathrm{dom}\, f, \mathrm{cod}\, f\rangle, x\rangle$.

Let us observe that there exists a strict category which has triple-like morphisms.

One can prove the following proposition

(2) Let C be a category structure with triple-like morphisms and f be a morphism of C. Then $\mathrm{dom}\, f = f_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, f = f_{\mathbf{1},\mathbf{2}}$ and $f = \langle\langle \mathrm{dom}\, f, \mathrm{cod}\, f\rangle, f_{\mathbf{2}}\rangle$.

Let C be a category structure with triple-like morphisms and let f be a morphism of C. Then $f_{\mathbf{1},\mathbf{1}}$ is an object of C. Then $f_{\mathbf{1},\mathbf{2}}$ is an object of C.

In this article we present several logical schemes. The scheme *CatEx* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a binary functor $\mathcal{F}$ yielding a set, and a ternary predicate $\mathcal{P}$, and states that:

There exists a strict category C with triple-like morphisms such that

(i) the objects of $C = \mathcal{A}$,

(ii) for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a, b, f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C,

(iii) for every morphism m of C there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathcal{P}[a, b, f]$, and

(iv) for all morphisms m_1, m_2 of C and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$

provided the parameters have the following properties:

- For all elements a, b, c of $\mathcal{A}$ and for all elements f, g of $\mathcal{B}$ such that $\mathcal{P}[a,b,f]$ and $\mathcal{P}[b,c,g]$ holds $\mathcal{F}(g,f) \in \mathcal{B}$ and $\mathcal{P}[a,c,\mathcal{F}(g,f)]$,
- Let a be an element of $\mathcal{A}$. Then there exists an element f of $\mathcal{B}$ such that
  (i) $\mathcal{P}[a,a,f]$, and
  (ii) for every element b of $\mathcal{A}$ and for every element g of $\mathcal{B}$ holds if $\mathcal{P}[a,b,g]$, then $\mathcal{F}(g,f) = g$ and if $\mathcal{P}[b,a,g]$, then $\mathcal{F}(f,g) = g$,
  and
- Let a, b, c, d be elements of $\mathcal{A}$ and f, g, h be elements of $\mathcal{B}$. If $\mathcal{P}[a,b,f]$ and $\mathcal{P}[b,c,g]$ and $\mathcal{P}[c,d,h]$, then $\mathcal{F}(h,\mathcal{F}(g,f)) = \mathcal{F}(\mathcal{F}(h,g),f)$.

The scheme *CatUniq* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a binary functor $\mathcal{F}$ yielding a set, and a ternary predicate $\mathcal{P}$, and states that:

Let C_1, C_2 be strict categories with triple-like morphisms. Suppose that the objects of $C_1 = \mathcal{A}$ and for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a,b,f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C_1 and for every morphism m of C_1 there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathcal{P}[a,b,f]$ and for all morphisms m_1, m_2 of C_1 and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$ and the objects of $C_2 = \mathcal{A}$ and for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a,b,f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C_2 and for every morphism m of C_2 there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathcal{P}[a,b,f]$ and for all morphisms m_1, m_2 of C_2 and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$. Then $C_1 = C_2$

provided the parameters have the following property:

- Let a be an element of $\mathcal{A}$. Then there exists an element f of $\mathcal{B}$ such that
  (i) $\mathcal{P}[a,a,f]$, and
  (ii) for every element b of $\mathcal{A}$ and for every element g of $\mathcal{B}$ holds if $\mathcal{P}[a,b,g]$, then $\mathcal{F}(g,f) = g$ and if $\mathcal{P}[b,a,g]$, then $\mathcal{F}(f,g) = g$.

The scheme *FunctorEx* deals with categories $\mathcal{A}$, $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an object of $\mathcal{B}$, and a unary functor $\mathcal{G}$ yielding a set, and states that:

There exists a functor F from $\mathcal{A}$ to $\mathcal{B}$ such that for every morphism f of $\mathcal{A}$ holds $F(f) = \mathcal{G}(f)$

provided the following conditions are met:

- Let f be a morphism of $\mathcal{A}$. Then $\mathcal{G}(f)$ is a morphism of $\mathcal{B}$ and for every morphism g of $\mathcal{B}$ such that $g = \mathcal{G}(f)$ holds $\operatorname{dom} g = \mathcal{F}(\operatorname{dom} f)$ and $\operatorname{cod} g = \mathcal{F}(\operatorname{cod} f)$,
- For every object a of $\mathcal{A}$ holds $\mathcal{G}(\mathrm{id}_a) = \mathrm{id}_{\mathcal{F}(a)}$, and
- For all morphisms f_1, f_2 of $\mathcal{A}$ and for all morphisms g_1, g_2 of $\mathcal{B}$ such that $g_1 = \mathcal{G}(f_1)$ and $g_2 = \mathcal{G}(f_2)$ and $\operatorname{dom} f_2 = \operatorname{cod} f_1$ holds $\mathcal{G}(f_2 \cdot f_1) = g_2 \cdot g_1$.

We now state two propositions:

(3) Let C_1 be a category and C_2 be a subcategory of C_1. Suppose C_1 is a subcategory of C_2. Then the category structure of $C_1 =$ the category structure of C_2.

(4) For every category C and for every subcategory D of C holds every subcategory of D is a subcategory of C.

Let C_1, C_2 be categories. Let us assume that there exists a category C such that C_1 is a subcategory of C and C_2 is a subcategory of C. And let us assume that there exists an object o_1 of C_1 such that o_1 is an object of C_2. The functor $C_1 \cap C_2$ yielding a strict category is defined by the conditions (Def. 2).

(Def. 2)(i) The objects of $C_1 \cap C_2 =$ (the objects of C_1) $\cap$ (the objects of C_2),

(ii) the morphisms of $C_1 \cap C_2 =$ (the morphisms of C_1) $\cap$ (the morphisms of C_2),

(iii) the dom-map of $C_1 \cap C_2 =$ (the dom-map of C_1) $\restriction$ (the morphisms of C_2),

(iv) the cod-map of $C_1 \cap C_2 =$ (the cod-map of C_1) $\restriction$ (the morphisms of C_2),

(v) the composition of $C_1 \cap C_2 =$ (the composition of C_1) $\restriction$ [: the morphisms of C_2, the morphisms of C_2 :], and

(vi) the id-map of $C_1 \cap C_2 =$ (the id-map of C_1) $\restriction$ (the objects of C_2).

In the sequel C is a category and C_1, C_2 are subcategories of C.
One can prove the following propositions:

(5) If the objects of C_1 meets the objects of C_2, then $C_1 \cap C_2 = C_2 \cap C_1$.

(6) Suppose the objects of C_1 meets the objects of C_2. Then $C_1 \cap C_2$ is a subcategory of C_1 and $C_1 \cap C_2$ is a subcategory of C_2.

Let C, D be categories and let F be a functor from C to D. The functor $\mathrm{Im} F$ yielding a strict subcategory of D is defined by the conditions (Def. 3).

(Def. 3)(i) The objects of $\mathrm{Im} F = \mathrm{rng}\,\mathrm{Obj}\, F$,

(ii) $\mathrm{rng}\, F \subseteq$ the morphisms of $\mathrm{Im} F$, and

(iii) for every subcategory E of D such that the objects of $E = \mathrm{rng}\,\mathrm{Obj}\, F$ and $\mathrm{rng}\, F \subseteq$ the morphisms of E holds $\mathrm{Im} F$ is a subcategory of E.

The following three propositions are true:

(7) Let C, D be categories, E be a subcategory of D, and F be a functor from C to D. If $\mathrm{rng}\, F \subseteq$ the morphisms of E, then F is a functor from C to E.

(8) For all categories C, D holds every functor F from C to D is a functor from C to $\mathrm{Im} F$.

(9) Let C, D be categories, E be a subcategory of D, F be a functor from C to E, and G be a functor from C to D. If $F = G$, then $\mathrm{Im} F = \mathrm{Im} G$.

2. Categorial Categories

Let I_1 be a set. We say that I_1 is categorial if and only if:

(Def. 4) For every set x such that $x \in I_1$ holds x is a category.

One can verify that there exists a non empty set which is categorial. Let X be a non empty set. Let us observe that X is categorial if and only if:

(Def. 5) Every element of X is a category.

Let X be a non empty categorial set. We see that the element of X is a category.
Let C be a category. We say that C is categorial if and only if the conditions (Def. 6) are satisfied.

(Def. 6)(i) The objects of C are categorial,

(ii) for every object a of C and for every category A such that $a = A$ holds $\mathrm{id}_a = \langle \langle A, A \rangle, \mathrm{id}_A \rangle$,

(iii) for every morphism m of C and for all categories A, B such that $A = \mathrm{dom}\, m$ and $B = \mathrm{cod}\, m$ there exists a functor F from A to B such that $m = \langle \langle A, B \rangle, F \rangle$, and

(iv) for all morphisms m_1, m_2 of C and for all categories A, B, C and for every functor F from A to B and for every functor G from B to C such that $m_1 = \langle \langle A, B \rangle, F \rangle$ and $m_2 = \langle \langle B, C \rangle, G \rangle$ holds $m_2 \cdot m_1 = \langle \langle A, C \rangle, G \cdot F \rangle$.

Let us note that every category which is categorial has also triple-like morphisms.
Next we state two propositions:

(10) Let C, D be categories. Suppose the category structure of $C =$ the category structure of D. If C is categorial, then D is categorial.

(11) For every category C holds $\dot{\circlearrowright}(C, \langle\langle C, C\rangle, \mathrm{id}_C\rangle)$ is categorial.

Let us observe that there exists a strict category which is categorial.
Next we state two propositions:

(12) For every categorial category C holds every object of C is a category.

(13) For every categorial category C and for every morphism f of C holds $\mathrm{dom}\, f = f_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, f = f_{\mathbf{1},\mathbf{2}}$.

Let C be a categorial category and let m be a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is a category. Then $m_{\mathbf{1},\mathbf{2}}$ is a category.
We now state the proposition

(14) Let C_1, C_2 be categorial categories. Suppose the objects of $C_1 =$ the objects of C_2 and the morphisms of $C_1 =$ the morphisms of C_2. Then the category structure of $C_1 =$ the category structure of C_2.

Let C be a categorial category. One can verify that every subcategory of C is categorial.
The following proposition is true

(15) Let C, D be categorial categories. Suppose the morphisms of $C \subseteq$ the morphisms of D. Then C is a subcategory of D.

Let a be a set. Let us assume that a is a category. The functor $\mathrm{cat}\, a$ yields a category and is defined as follows:

(Def. 7) $\mathrm{cat}\, a = a$.

The following proposition is true

(16) For every categorial category C and for every object c of C holds $\mathrm{cat}\, c = c$.

Let C be a categorial category and let m be a morphism of C. Then $m_{\mathbf{2}}$ is a functor from $\mathrm{cat}\,\mathrm{dom}\, m$ to $\mathrm{cat}\,\mathrm{cod}\, m$.
Next we state two propositions:

(17) Let X be a categorial non empty set and Y be a non empty set. Suppose that
(i) for all elements A, B, C of X and for every functor F from A to B and for every functor G from B to C such that $F \in Y$ and $G \in Y$ holds $G \cdot F \in Y$, and
(ii) for every element A of X holds $\mathrm{id}_A \in Y$.

Then there exists a strict categorial category C such that
(iii) the objects of $C = X$, and
(iv) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C iff $F \in Y$.

(18) Let X be a categorial non empty set, Y be a non empty set, and C_1, C_2 be strict categorial categories. Suppose that
(i) the objects of $C_1 = X$,
(ii) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C_1 iff $F \in Y$,
(iii) the objects of $C_2 = X$, and
(iv) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C_2 iff $F \in Y$.

Then $C_1 = C_2$.

Let I_1 be a categorial category. We say that I_1 is full if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let a, b be categories. Suppose a is an object of I_1 and b is an object of I_1. Let F be a functor from a to b. Then $\langle\langle a, b\rangle, F\rangle$ is a morphism of I_1.

Let us note that there exists a categorial strict category which is full.

The following four propositions are true:

(19) Let C_1, C_2 be full categorial categories. Suppose the objects of $C_1 =$ the objects of C_2. Then the category structure of $C_1 =$ the category structure of C_2.

(20) For every categorial non empty set A there exists a full categorial strict category C such that the objects of $C = A$.

(21) Let C be a categorial category and D be a full categorial category. Suppose the objects of $C \subseteq$ the objects of D. Then C is a subcategory of D.

(22) Let C be a category, D_1, D_2 be categorial categories, F_1 be a functor from C to D_1, and F_2 be a functor from C to D_2. If $F_1 = F_2$, then $\mathrm{Im} F_1 = \mathrm{Im} F_2$.

3. Slice Categories

Let C be a category and let o be an object of C. The functor $\mathrm{Hom}(o)$ yielding a subset of the morphisms of C is defined as follows:

(Def. 9) $\mathrm{Hom}(o) = (\text{the cod-map of } C)^{-1}(\{o\})$.

The functor $\mathrm{hom}(o,\square)$ yields a subset of the morphisms of C and is defined by:

(Def. 10) $\mathrm{hom}(o,\square) = (\text{the dom-map of } C)^{-1}(\{o\})$.

Let C be a category and let o be an object of C. Observe that $\mathrm{Hom}(o)$ is non empty and $\mathrm{hom}(o,\square)$ is non empty.

Next we state several propositions:

(23) For every category C and for every object a of C and for every morphism f of C holds $f \in \mathrm{Hom}(a)$ iff $\mathrm{cod}\, f = a$.

(24) For every category C and for every object a of C and for every morphism f of C holds $f \in \mathrm{hom}(a,\square)$ iff $\mathrm{dom}\, f = a$.

(25) For every category C and for all objects a, b of C holds $\mathrm{hom}(a,b) = \mathrm{hom}(a,\square) \cap \mathrm{Hom}(b)$.

(26) For every category C and for every morphism f of C holds $f \in \mathrm{hom}(\mathrm{dom}\, f,\square)$ and $f \in \mathrm{Hom}(\mathrm{cod}\, f)$.

(27) For every category C and for every morphism f of C and for every element g of $\mathrm{Hom}(\mathrm{dom}\, f)$ holds $f \cdot g \in \mathrm{Hom}(\mathrm{cod}\, f)$.

(28) For every category C and for every morphism f of C and for every element g of $\mathrm{hom}(\mathrm{cod}\, f,\square)$ holds $g \cdot f \in \mathrm{hom}(\mathrm{dom}\, f,\square)$.

Let C be a category and let o be an object of C. The functor $\mathrm{SliceCat}(C,o)$ yields a strict category with triple-like morphisms and is defined by the conditions (Def. 11).

(Def. 11)(i) The objects of $\mathrm{SliceCat}(C,o) = \mathrm{Hom}(o)$,

(ii) for all elements a, b of $\mathrm{Hom}(o)$ and for every morphism f of C such that $\mathrm{dom}\, b = \mathrm{cod}\, f$ and $a = b \cdot f$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of $\mathrm{SliceCat}(C,o)$,

(iii) for every morphism m of $\mathrm{SliceCat}(C,o)$ there exist elements a, b of $\mathrm{Hom}(o)$ and there exists a morphism f of C such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathrm{dom}\, b = \mathrm{cod}\, f$ and $a = b \cdot f$, and

(iv) for all morphisms m_1, m_2 of $\mathrm{SliceCat}(C,o)$ and for all elements a_1, a_2, a_3 of $\mathrm{Hom}(o)$ and for all morphisms f_1, f_2 of C such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, f_2 \cdot f_1\rangle$.

The functor $\mathrm{SliceCat}(o,C)$ yielding a strict category with triple-like morphisms is defined by the conditions (Def. 12).

(Def. 12)(i) The objects of $\mathrm{SliceCat}(o,C) = \mathrm{hom}(o,\square)$,

(ii) for all elements a, b of $\mathrm{hom}(o,\square)$ and for every morphism f of C such that $\mathrm{dom}\, f = \mathrm{cod}\, a$ and $f \cdot a = b$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of $\mathrm{SliceCat}(o,C)$,

(iii) for every morphism m of $\mathrm{SliceCat}(o,C)$ there exist elements a, b of $\mathrm{hom}(o,\square)$ and there exists a morphism f of C such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathrm{dom}\, f = \mathrm{cod}\, a$ and $f \cdot a = b$, and

(iv) for all morphisms m_1, m_2 of $\mathrm{SliceCat}(o,C)$ and for all elements a_1, a_2, a_3 of $\mathrm{hom}(o,\square)$ and for all morphisms f_1, f_2 of C such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, f_2 \cdot f_1\rangle$.

Let C be a category, let o be an object of C, and let m be a morphism of $\mathrm{SliceCat}(C,o)$. Then $m_{\mathbf{2}}$ is a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is an element of $\mathrm{Hom}(o)$. Then $m_{\mathbf{1},\mathbf{2}}$ is an element of $\mathrm{Hom}(o)$.

The following two propositions are true:

(29) Let C be a category, a be an object of C, and m be a morphism of $\mathrm{SliceCat}(C,a)$. Then $m = \langle\langle m_{\mathbf{1},\mathbf{1}}, m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$ and $\mathrm{dom}(m_{\mathbf{1},\mathbf{2}}) = \mathrm{cod}(m_{\mathbf{2}})$ and $m_{\mathbf{1},\mathbf{1}} = m_{\mathbf{1},\mathbf{2}} \cdot m_{\mathbf{2}}$ and $\mathrm{dom}\, m = m_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, m = m_{\mathbf{1},\mathbf{2}}$.

(30) Let C be a category, o be an object of C, f be an element of $\mathrm{Hom}(o)$, and a be an object of $\mathrm{SliceCat}(C,o)$. If $a = f$, then $\mathrm{id}_a = \langle\langle a, a\rangle, \mathrm{id}_{\mathrm{dom}\, f}\rangle$.

Let C be a category, let o be an object of C, and let m be a morphism of $\mathrm{SliceCat}(o,C)$. Then $m_{\mathbf{2}}$ is a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is an element of $\mathrm{hom}(o,\square)$. Then $m_{\mathbf{1},\mathbf{2}}$ is an element of $\mathrm{hom}(o,\square)$.

One can prove the following two propositions:

(31) Let C be a category, a be an object of C, and m be a morphism of $\mathrm{SliceCat}(a,C)$. Then $m = \langle\langle m_{\mathbf{1},\mathbf{1}}, m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$ and $\mathrm{dom}(m_{\mathbf{2}}) = \mathrm{cod}(m_{\mathbf{1},\mathbf{1}})$ and $m_{\mathbf{2}} \cdot m_{\mathbf{1},\mathbf{1}} = m_{\mathbf{1},\mathbf{2}}$ and $\mathrm{dom}\, m = m_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, m = m_{\mathbf{1},\mathbf{2}}$.

(32) Let C be a category, o be an object of C, f be an element of $\mathrm{hom}(o,\square)$, and a be an object of $\mathrm{SliceCat}(o,C)$. If $a = f$, then $\mathrm{id}_a = \langle\langle a, a\rangle, \mathrm{id}_{\mathrm{cod}\, f}\rangle$.

4. Functors Between Slice Categories

Let C be a category and let f be a morphism of C. The functor $\mathrm{SliceFunctor}(f)$ yields a functor from $\mathrm{SliceCat}(C, \mathrm{dom}\, f)$ to $\mathrm{SliceCat}(C, \mathrm{cod}\, f)$ and is defined by:

(Def. 13) For every morphism m of $\mathrm{SliceCat}(C, \mathrm{dom}\, f)$ holds $(\mathrm{SliceFunctor}(f))(m) = \langle\langle f \cdot m_{\mathbf{1},\mathbf{1}}, f \cdot m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$.

The functor $\mathrm{SliceContraFunctor}(f)$ yielding a functor from $\mathrm{SliceCat}(\mathrm{cod}\, f, C)$ to $\mathrm{SliceCat}(\mathrm{dom}\, f, C)$ is defined by:

(Def. 14) For every morphism m of $\mathrm{SliceCat}(\mathrm{cod}\, f, C)$ holds $(\mathrm{SliceContraFunctor}(f))(m) = \langle\langle m_{\mathbf{1},\mathbf{1}} \cdot f, m_{\mathbf{1},\mathbf{2}} \cdot f\rangle, m_{\mathbf{2}}\rangle$.

Next we state two propositions:

(33) For every category C and for all morphisms f, g of C such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{SliceFunctor}(g \cdot f) = \mathrm{SliceFunctor}(g) \cdot \mathrm{SliceFunctor}(f)$.

(34) For every category C and for all morphisms f, g of C such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{SliceContraFunctor}(g \cdot f) = \mathrm{SliceContraFunctor}(f) \cdot \mathrm{SliceContraFunctor}(g)$.

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Received October 24, 1994

Published January 2, 2004