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Propositional Calculus for Boolean Valued Functions. Part V

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [3], [5], [4], [2], and [1] provide the notation and terminology for this paper.

We follow the rules: Y is a non empty set and a, b, c, d, e, f, g are elements of $Boolean^Y$.

We now state a number of propositions:

(1) $(a \vee b) \wedge (b \Rightarrow c) \Subset a \vee c$.

(2) $a \wedge (a \Rightarrow b) \Subset b$.

(3) $(a \Rightarrow b) \wedge \neg b \Subset \neg a$.

(4) $(a \vee b) \wedge \neg a \Subset b$.

(5) $(a \Rightarrow b) \wedge (\neg a \Rightarrow b) \Subset b$.

(6) $(a \Rightarrow b) \wedge (a \Rightarrow \neg b) \Subset \neg a$.

(7) $a \Rightarrow b \wedge c \Subset a \Rightarrow b$.

(8) $a \vee b \Rightarrow c \Subset a \Rightarrow c$.

(9) $a \Rightarrow b \Subset a \wedge c \Rightarrow b$.

(10) $a \Rightarrow b \Subset a \wedge c \Rightarrow b \wedge c$.

(11) $a \Rightarrow b \Subset a \Rightarrow b \vee c$.

(12) $a \Rightarrow b \Subset a \vee c \Rightarrow b \vee c$.

(13) $a \wedge b \vee c \Subset a \vee c$.

(14) $a \wedge b \vee c \wedge d \Subset a \vee c$.

(15) $(a \vee b) \wedge (b \Rightarrow c) \Subset a \vee c$.

(16) $(a \Rightarrow b) \wedge (\neg a \Rightarrow c) \Subset b \vee c$.

(17) $(a \Rightarrow c) \wedge (b \Rightarrow \neg c) \Subset \neg a \vee \neg b$.

(18) $(a \vee b) \wedge (\neg a \vee c) \Subset b \vee c$.

(19) $(a \Rightarrow b) \wedge (c \Rightarrow d) \Subset a \wedge c \Rightarrow b \wedge d$.

(20) $(a \Rightarrow b) \wedge (a \Rightarrow c) \Subset a \Rightarrow b \wedge c$.

(21) $(a \Rightarrow c) \wedge (b \Rightarrow c) \Subset a \vee b \Rightarrow c$.

(22) $(a \Rightarrow b) \wedge (c \Rightarrow d) \Subset a \vee c \Rightarrow b \vee d$.

(23) $(a \Rightarrow b) \wedge (a \Rightarrow c) \Subset a \Rightarrow b \vee c$.

(24) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of $Boolean^Y$ holds $(b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \Subset a_2 \Rightarrow a_1$.

(25) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of $Boolean^Y$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \wedge \neg(b_2 \wedge c_2) \Subset (a_2 \Rightarrow a_1) \wedge (b_2 \Rightarrow b_1) \wedge (c_2 \Rightarrow c_1)$.

(26) For all elements a_1, b_1, a_2, b_2 of $Boolean^Y$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge \neg(a_2 \wedge b_2) \Rightarrow \neg(a_1 \wedge b_1) = true(Y)$.

(27) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of $Boolean^Y$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \wedge \neg(b_2 \wedge c_2) \Subset \neg(a_1 \wedge b_1) \wedge \neg(a_1 \wedge c_1) \wedge \neg(b_1 \wedge c_1)$.

(28) $a \wedge b \Subset a$.

(29) $a \wedge b \wedge c \Subset a$ and $a \wedge b \wedge c \Subset b$.

(30) $a \wedge b \wedge c \wedge d \Subset a$ and $a \wedge b \wedge c \wedge d \Subset b$.

(31) $a \wedge b \wedge c \wedge d \wedge e \Subset a$ and $a \wedge b \wedge c \wedge d \wedge e \Subset b$.

(32) $a \wedge b \wedge c \wedge d \wedge e \wedge f \Subset a$ and $a \wedge b \wedge c \wedge d \wedge e \wedge f \Subset b$.

(33) $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g \Subset a$ and $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g \Subset b$.

(34) If $a \Subset b$ and $c \Subset d$, then $a \wedge c \Subset b \wedge d$.

(35) If $a \wedge b \Subset c$, then $a \wedge \neg c \Subset \neg b$.

(36) $(a \Rightarrow c) \wedge (b \Rightarrow c) \wedge (a \vee b) \Subset c$.

(37) $((a \Rightarrow c) \vee (b \Rightarrow c)) \wedge (a \wedge b) \Subset c$.

(38) If $a \Subset b$ and $c \Subset d$, then $a \vee c \Subset b \vee d$.

(39) $a \Subset a \vee b$.

(40) $a \wedge b \Subset a \vee b$.

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