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Propositional Calculus for Boolean Valued Functions. Part IV

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [3], [5], [8], [7], [6], [1], [4], and [2] provide the notation and terminology for this paper.

In this paper Y is a non empty set.

Next we state a number of propositions:

(1) For all elements a, b, c, d of $Boolean^Y$ holds $a \Rightarrow b \wedge c \wedge d = (a \Rightarrow b) \wedge (a \Rightarrow c) \wedge (a \Rightarrow d)$.

(2) For all elements a, b, c, d of $Boolean^Y$ holds $a \Rightarrow b \vee c \vee d = (a \Rightarrow b) \vee (a \Rightarrow c) \vee (a \Rightarrow d)$.

(3) For all elements a, b, c, d of $Boolean^Y$ holds $a \wedge b \wedge c \Rightarrow d = (a \Rightarrow d) \vee (b \Rightarrow d) \vee (c \Rightarrow d)$.

(4) For all elements a, b, c, d of $Boolean^Y$ holds $a \vee b \vee c \Rightarrow d = (a \Rightarrow d) \wedge (b \Rightarrow d) \wedge (c \Rightarrow d)$.

(5) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) = (a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) \wedge (b \Rightarrow a) \wedge (a \Rightarrow c)$.

(6) For all elements a, b of $Boolean^Y$ holds $a = a \wedge b \vee a \wedge \neg b$.

(7) For all elements a, b of $Boolean^Y$ holds $a = (a \vee b) \wedge (a \vee \neg b)$.

(8) For all elements a, b, c of $Boolean^Y$ holds $a = a \wedge b \wedge c \vee a \wedge b \wedge \neg c \vee a \wedge \neg b \wedge c \vee a \wedge \neg b \wedge \neg c$.

(9) For all elements a, b, c of $Boolean^Y$ holds $a = (a \vee b \vee c) \wedge (a \vee b \vee \neg c) \wedge (a \vee \neg b \vee c) \wedge (a \vee \neg b \vee \neg c)$.

(10) For all elements a, b of $Boolean^Y$ holds $a \wedge b = a \wedge (\neg a \vee b)$.

(11) For all elements a, b of $Boolean^Y$ holds $a \vee b = a \vee \neg a \wedge b$.

(12) For all elements a, b of $Boolean^Y$ holds $a \oplus b = \neg(a \Leftrightarrow b)$.

(13) For all elements a, b of $Boolean^Y$ holds $a \oplus b = (a \vee b) \wedge (\neg a \vee \neg b)$.

(14) For every element a of $Boolean^Y$ holds $a \oplus true(Y) = \neg a$.

(15) For every element a of $Boolean^Y$ holds $a \oplus false(Y) = a$.

(16) For all elements a, b of $Boolean^Y$ holds $a \oplus b = \neg a \oplus \neg b$.

(17) For all elements a, b of $Boolean^Y$ holds $\neg(a \oplus b) = a \oplus \neg b$.

(18) For all elements a, b of $Boolean^Y$ holds $a \Leftrightarrow b = (a \vee \neg b) \wedge (\neg a \vee b)$.

(19) For all elements a, b of $Boolean^Y$ holds $a \Leftrightarrow b = a \wedge b \vee \neg a \wedge \neg b$.

(20) For every element a of $Boolean^Y$ holds $a \Leftrightarrow true(Y) = a$.

(21) For every element a of $Boolean^Y$ holds $a \Leftrightarrow false(Y) = \neg a$.

(22) For all elements a, b of $Boolean^Y$ holds $\neg(a \Leftrightarrow b) = a \Leftrightarrow \neg b$.

(23) For all elements a, b of $Boolean^Y$ holds $\neg a \Subset a \Rightarrow b \Leftrightarrow \neg a$.

(24) For all elements a, b of $Boolean^Y$ holds $\neg a \Subset b \Rightarrow a \Leftrightarrow \neg b$.

(25) For all elements a, b of $Boolean^Y$ holds $a \Subset a \vee b \Leftrightarrow b \vee a \Leftrightarrow a$.

(26) For every element a of $Boolean^Y$ holds $a \Rightarrow \neg a \Leftrightarrow \neg a = true(Y)$.

(27) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow a \Rightarrow a = true(Y)$.

(28) For all elements a, b, c, d of $Boolean^Y$ holds $(a \Rightarrow c) \wedge (b \Rightarrow d) \wedge (\neg c \vee \neg d) \Rightarrow \neg a \vee \neg b = true(Y)$.

(29) For all elements a, b, c of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow a \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow c = true(Y)$.

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