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Propositional Calculus for Boolean Valued Functions. Part II

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [3], [2], [5], [4], and [1] provide the notation and terminology for this paper.

In this paper Y is a non empty set.

The following propositions are true:

(1) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Rightarrow b \Rightarrow a \wedge b = \mathit{true}(Y)$.

(2) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Rightarrow b \Rightarrow b \Rightarrow a \Rightarrow a \Leftrightarrow b = \mathit{true}(Y)$.

(3) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \vee b \Leftrightarrow b \vee a = \mathit{true}(Y)$.

(4) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $a \wedge b \Rightarrow c \Rightarrow a \Rightarrow b \Rightarrow c = \mathit{true}(Y)$.

(5) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $a \Rightarrow b \Rightarrow c \Rightarrow a \wedge b \Rightarrow c = \mathit{true}(Y)$.

(6) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $c \Rightarrow a \Rightarrow c \Rightarrow b \Rightarrow c \Rightarrow a \wedge b = \mathit{true}(Y)$.

(7) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $a \vee b \Rightarrow c \Rightarrow (a \Rightarrow c) \vee (b \Rightarrow c) = \mathit{true}(Y)$.

(8) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $a \Rightarrow c \Rightarrow b \Rightarrow c \Rightarrow a \vee b \Rightarrow c = \mathit{true}(Y)$.

(9) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $(a \Rightarrow c) \wedge (b \Rightarrow c) \Rightarrow a \vee b \Rightarrow c = \mathit{true}(Y)$.

(10) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Rightarrow b \wedge \neg b \Rightarrow \neg a = \mathit{true}(Y)$.

(11) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $(a \vee b) \wedge (a \vee c) \Rightarrow a \vee b \wedge c = \mathit{true}(Y)$.

(12) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $a \wedge (b \vee c) \Rightarrow a \wedge b \vee a \wedge c = \mathit{true}(Y)$.

(13) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $(a \vee c) \wedge (b \vee c) \Rightarrow a \wedge b \vee c = \mathit{true}(Y)$.

(14) For all elements a, b, c of $\mathit{Boolean}^Y$ holds $(a \vee b) \wedge c \Rightarrow a \wedge c \vee b \wedge c = \mathit{true}(Y)$.

(15) For all elements a, b of $\mathit{Boolean}^Y$ such that $a \wedge b = \mathit{true}(Y)$ holds $a \vee b = \mathit{true}(Y)$.

(16) For all elements a, b, c of $\mathit{Boolean}^Y$ such that $a \Rightarrow b = \mathit{true}(Y)$ holds $a \vee c \Rightarrow b \vee c = \mathit{true}(Y)$.

(17) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ holds $a \wedge c \Rightarrow b \wedge c = true(Y)$.

(18) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow a = true(Y)$ and $c \Rightarrow b = true(Y)$ holds $c \Rightarrow a \wedge b = true(Y)$.

(19) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow c = true(Y)$ and $b \Rightarrow c = true(Y)$ holds $a \vee b \Rightarrow c = true(Y)$.

(20) For all elements a, b of $Boolean^Y$ such that $a \vee b = true(Y)$ and $\neg a = true(Y)$ holds $b = true(Y)$.

(21) For all elements a, b, c, d of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ and $c \Rightarrow d = true(Y)$ holds $a \wedge c \Rightarrow b \wedge d = true(Y)$.

(22) For all elements a, b, c, d of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ and $c \Rightarrow d = true(Y)$ holds $a \vee c \Rightarrow b \vee d = true(Y)$.

(23) For all elements a, b of $Boolean^Y$ such that $a \wedge \neg b \Rightarrow \neg a = true(Y)$ holds $a \Rightarrow b = true(Y)$.

(25)[1] For all elements a, b of $Boolean^Y$ such that $a \Rightarrow \neg b = true(Y)$ holds $b \Rightarrow \neg a = true(Y)$.

(26) For all elements a, b of $Boolean^Y$ such that $\neg a \Rightarrow b = true(Y)$ holds $\neg b \Rightarrow a = true(Y)$.

(27) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow a \vee b = true(Y)$.

(28) For all elements a, b of $Boolean^Y$ holds $a \vee b \Rightarrow \neg a \Rightarrow b = true(Y)$.

(29) For all elements a, b of $Boolean^Y$ holds $\neg(a \vee b) \Rightarrow \neg a \wedge \neg b = true(Y)$.

(30) For all elements a, b of $Boolean^Y$ holds $\neg a \wedge \neg b \Rightarrow \neg(a \vee b) = true(Y)$.

(31) For all elements a, b of $Boolean^Y$ holds $\neg(a \vee b) \Rightarrow \neg a = true(Y)$.

(32) For every element a of $Boolean^Y$ holds $a \vee a \Rightarrow a = true(Y)$.

(33) For all elements a, b of $Boolean^Y$ holds $a \wedge \neg a \Rightarrow b = true(Y)$.

(34) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow \neg a \vee b = true(Y)$.

(35) For all elements a, b of $Boolean^Y$ holds $a \wedge b \Rightarrow \neg(a \Rightarrow \neg b) = true(Y)$.

(36) For all elements a, b of $Boolean^Y$ holds $\neg(a \Rightarrow \neg b) \Rightarrow a \wedge b = true(Y)$.

(37) For all elements a, b of $Boolean^Y$ holds $\neg(a \wedge b) \Rightarrow \neg a \vee \neg b = true(Y)$.

(38) For all elements a, b of $Boolean^Y$ holds $\neg a \vee \neg b \Rightarrow \neg(a \wedge b) = true(Y)$.

(39) For all elements a, b of $Boolean^Y$ holds $a \wedge b \Rightarrow a = true(Y)$.

(40) For all elements a, b of $Boolean^Y$ holds $a \wedge b \Rightarrow a \vee b = true(Y)$.

(41) For all elements a, b of $Boolean^Y$ holds $a \wedge b \Rightarrow b = true(Y)$.

(42) For every element a of $Boolean^Y$ holds $a \Rightarrow a \wedge a = true(Y)$.

(43) For all elements a, b of $Boolean^Y$ holds $a \Leftrightarrow b \Rightarrow a \Rightarrow b = true(Y)$.

(44) For all elements a, b of $Boolean^Y$ holds $a \Leftrightarrow b \Rightarrow b \Rightarrow a = true(Y)$.

(45) For all elements a, b, c of $Boolean^Y$ holds $a \vee b \vee c \Rightarrow a \vee (b \vee c) = true(Y)$.

(46) For all elements a, b, c of $Boolean^Y$ holds $a \wedge b \wedge c \Rightarrow a \wedge (b \wedge c) = true(Y)$.

(47) For all elements a, b, c of $Boolean^Y$ holds $a \vee (b \vee c) \Rightarrow a \vee b \vee c = true(Y)$.

[1] The proposition (24) has been removed.

References

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