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Propositional Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [3], [5], [8], [7], [6], [1], [4], and [2] provide the notation and terminology for this paper.

In this paper Y denotes a non empty set.

We now state a number of propositions:

(1) For all elements a, b of $Boolean^Y$ holds $a = true(Y)$ and $b = true(Y)$ iff $a \wedge b = true(Y)$.

(2) For all elements a, b of $Boolean^Y$ such that $a = true(Y)$ and $a \Rightarrow b = true(Y)$ holds $b = true(Y)$.

(3) For all elements a, b of $Boolean^Y$ such that $a = true(Y)$ holds $a \vee b = true(Y)$.

(5)[1] For all elements a, b of $Boolean^Y$ such that $b = true(Y)$ holds $a \Rightarrow b = true(Y)$.

(6) For all elements a, b of $Boolean^Y$ such that $\neg a = true(Y)$ holds $a \Rightarrow b = true(Y)$.

(7) For every element a of $Boolean^Y$ holds $\neg(a \wedge \neg a) = true(Y)$.

(8) For every element a of $Boolean^Y$ holds $a \Rightarrow a = true(Y)$.

(9) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow b = true(Y)$ iff $\neg b \Rightarrow \neg a = true(Y)$.

(10) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ and $b \Rightarrow c = true(Y)$ holds $a \Rightarrow c = true(Y)$.

(11) For all elements a, b of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ and $a \Rightarrow \neg b = true(Y)$ holds $\neg a = true(Y)$.

(12) For every element a of $Boolean^Y$ holds $\neg a \Rightarrow a \Rightarrow a = true(Y)$.

(13) For all elements a, b, c of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow \neg(b \wedge c) \Rightarrow \neg(a \wedge c) = true(Y)$.

(14) For all elements a, b, c of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow c = true(Y)$.

[1] The proposition (4) has been removed.

(15) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b = true(Y)$ holds $b \Rightarrow c \Rightarrow a \Rightarrow c = true(Y)$.

(16) For all elements a, b of $Boolean^Y$ holds $b \Rightarrow a \Rightarrow b = true(Y)$.

(17) For all elements a, b, c of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow c \Rightarrow b \Rightarrow c = true(Y)$.

(18) For all elements a, b of $Boolean^Y$ holds $b \Rightarrow b \Rightarrow a \Rightarrow a = true(Y)$.

(19) For all elements a, b, c of $Boolean^Y$ holds $c \Rightarrow b \Rightarrow a \Rightarrow b \Rightarrow c \Rightarrow a = true(Y)$.

(20) For all elements a, b, c of $Boolean^Y$ holds $b \Rightarrow c \Rightarrow a \Rightarrow b \Rightarrow a \Rightarrow c = true(Y)$.

(21) For all elements a, b, c of $Boolean^Y$ holds $b \Rightarrow b \Rightarrow c \Rightarrow b \Rightarrow c = true(Y)$.

(22) For all elements a, b, c of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow b \Rightarrow a \Rightarrow c = true(Y)$.

(23) For all elements a, b of $Boolean^Y$ such that $a = true(Y)$ holds $a \Rightarrow b \Rightarrow b = true(Y)$.

(24) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow b \Rightarrow a = true(Y)$ holds $b \Rightarrow c \Rightarrow a = true(Y)$.

(25) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow b \Rightarrow a = true(Y)$ and $b = true(Y)$ holds $c \Rightarrow a = true(Y)$.

(26) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow b \Rightarrow a = true(Y)$ and $b = true(Y)$ and $c = true(Y)$ holds $a = true(Y)$.

(27) For all elements b, c of $Boolean^Y$ such that $b \Rightarrow b \Rightarrow c = true(Y)$ holds $b \Rightarrow c = true(Y)$.

(28) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b \Rightarrow c = true(Y)$ holds $a \Rightarrow b \Rightarrow a \Rightarrow c = true(Y)$.

(29) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b \Rightarrow c = true(Y)$ and $a \Rightarrow b = true(Y)$ holds $a \Rightarrow c = true(Y)$.

(30) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow b \Rightarrow c = true(Y)$ and $a \Rightarrow b = true(Y)$ and $a = true(Y)$ holds $c = true(Y)$.

(31) For all elements a, b, c, d of $Boolean^Y$ such that $a \Rightarrow b \Rightarrow c = true(Y)$ and $a \Rightarrow c \Rightarrow d = true(Y)$ holds $a \Rightarrow b \Rightarrow d = true(Y)$.

(32) For all elements a, b of $Boolean^Y$ holds $\neg a \Rightarrow \neg b \Rightarrow b \Rightarrow a = true(Y)$.

(33) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow b \Rightarrow \neg b \Rightarrow \neg a = true(Y)$.

(34) For all elements a, b of $Boolean^Y$ holds $a \Rightarrow \neg b \Rightarrow b \Rightarrow \neg a = true(Y)$.

(35) For all elements a, b of $Boolean^Y$ holds $\neg a \Rightarrow b \Rightarrow \neg b \Rightarrow a = true(Y)$.

(36) For every element a of $Boolean^Y$ holds $a \Rightarrow \neg a \Rightarrow \neg a = true(Y)$.

(37) For all elements a, b of $Boolean^Y$ holds $\neg a \Rightarrow a \Rightarrow b = true(Y)$.

(38) For all elements a, b, c of $Boolean^Y$ holds $\neg(a \wedge b \wedge c) = \neg a \vee \neg b \vee \neg c$.

(39) For all elements a, b, c of $Boolean^Y$ holds $\neg(a \vee b \vee c) = \neg a \wedge \neg b \wedge \neg c$.

(40) For all elements a, b, c, d of $Boolean^Y$ holds $a \vee b \wedge c \wedge d = (a \vee b) \wedge (a \vee c) \wedge (a \vee d)$.

(41) For all elements a, b, c, d of $Boolean^Y$ holds $a \wedge (b \vee c \vee d) = a \wedge b \vee a \wedge c \vee a \wedge d$.

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