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Predicate Calculus for Boolean Valued Functions. Part II

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Summary. In this paper, we have proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [5], [7], [10], [9], [8], [1], [6], [4], [2], and [3] provide the notation and terminology for this paper.

1. Preliminaries

In this paper Y is a non empty set.

The following propositions are true:

(1) For all elements a, b, c of $\mathit{Boolean}^Y$ such that $a \Subset b \Rightarrow c$ holds $a \wedge b \Subset c$.

(2) For all elements a, b, c of $\mathit{Boolean}^Y$ such that $a \wedge b \Subset c$ holds $a \Subset b \Rightarrow c$.

(3) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \vee a \wedge b = a$.

(4) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \wedge (a \vee b) = a$.

(5) For every element a of $\mathit{Boolean}^Y$ holds $a \wedge \neg a = \mathit{false}(Y)$.

(6) For every element a of $\mathit{Boolean}^Y$ holds $a \vee \neg a = \mathit{true}(Y)$.

(7) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a)$.

(8) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Rightarrow b = \neg a \vee b$.

(9) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \oplus b = \neg a \wedge b \vee a \wedge \neg b$.

(10) For all elements a, b of $\mathit{Boolean}^Y$ holds $a \Leftrightarrow b = \mathit{true}(Y)$ iff $a \Rightarrow b = \mathit{true}(Y)$ and $b \Rightarrow a = \mathit{true}(Y)$.

(11) For all elements a, b, c of $\mathit{Boolean}^Y$ such that $a \Leftrightarrow b = \mathit{true}(Y)$ and $b \Leftrightarrow c = \mathit{true}(Y)$ holds $a \Leftrightarrow c = \mathit{true}(Y)$.

(12) For all elements a, b of $\mathit{Boolean}^Y$ such that $a \Leftrightarrow b = \mathit{true}(Y)$ holds $\neg a \Leftrightarrow \neg b = \mathit{true}(Y)$.

(13) For all elements a, b, c, d of $Boolean^Y$ such that $a \Leftrightarrow b = true(Y)$ and $c \Leftrightarrow d = true(Y)$ holds $a \wedge c \Leftrightarrow b \wedge d = true(Y)$.

(14) For all elements a, b, c, d of $Boolean^Y$ such that $a \Leftrightarrow b = true(Y)$ and $c \Leftrightarrow d = true(Y)$ holds $a \Rightarrow c \Leftrightarrow b \Rightarrow d = true(Y)$.

(15) For all elements a, b, c, d of $Boolean^Y$ such that $a \Leftrightarrow b = true(Y)$ and $c \Leftrightarrow d = true(Y)$ holds $a \vee c \Leftrightarrow b \vee d = true(Y)$.

(16) For all elements a, b, c, d of $Boolean^Y$ such that $a \Leftrightarrow b = true(Y)$ and $c \Leftrightarrow d = true(Y)$ holds $a \Leftrightarrow c \Leftrightarrow b \Leftrightarrow d = true(Y)$.

2. Predicate Calculus

We now state a number of propositions:

(17) Let a, b be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Then $\forall_{a \Leftrightarrow b, P_1} G = \forall_{a \Rightarrow b, P_1} G \wedge \forall_{b \Rightarrow a, P_1} G$.

(18) For every element a of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for all partitions P_1, P_2 of Y holds $\forall_{a,P_1} G \Subset \exists_{a,P_2} G$.

(19) Let a, u be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. Suppose G is independent and $P_1 \in G$ and u is independent of P_1, G. If $a \Rightarrow u = true(Y)$, then $\forall_{a,P_1} G \Rightarrow u = true(Y)$.

(20) Let u be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. If u is independent of P_1, G, then $\exists_{u,P_1} G \Subset u$.

(21) Let u be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. If u is independent of P_1, G, then $u \Subset \forall_{u,P_1} G$.

(22) Let u be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1, P_2 be partitions of Y. If u is independent of P_2, G, then $\forall_{u,P_1} G \Subset \forall_{u,P_2} G$.

(23) Let u be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1, P_2 be partitions of Y. If u is independent of P_1, G, then $\exists_{u,P_1} G \Subset \exists_{u,P_2} G$.

(24) For all elements a, b of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for every partition P_1 of Y holds $\forall_{a \Leftrightarrow b, P_1} G \Subset \forall_{a,P_1} G \Leftrightarrow \forall_{b,P_1} G$.

(25) For all elements a, b of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for every partition P_1 of Y holds $\forall_{a \wedge b, P_1} G \Subset a \wedge \forall_{b,P_1} G$.

(26) For all elements a, u of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for every partition P_1 of Y holds $\forall_{a,P_1} G \Rightarrow u \Subset \exists_{a \Rightarrow u, P_1} G$.

(27) Let a, b be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. If $a \Leftrightarrow b = true(Y)$, then $\forall_{a,P_1} G \Leftrightarrow \forall_{b,P_1} G = true(Y)$.

(28) Let a, b be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P_1 be a partition of Y. If $a \Leftrightarrow b = true(Y)$, then $\exists_{a,P_1} G \Leftrightarrow \exists_{b,P_1} G = true(Y)$.

References

[1] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[2] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_1.html.

[3] Shunichi Kobayashi and Yatsuka Nakamura. A theory of Boolean valued functions and quantifiers with respect to partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_2.html.

[4] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/eqrel_1.html.

[5] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[6] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fraenkel.html.

[7] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[8] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[9] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/valuat_1.html.

[10] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/margrel1.html.

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