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Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we have proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [5], [7], [6], [4], [3], [1], and [2] provide the notation and terminology for this paper.

For simplicity, we use the following convention: Y denotes a non empty set, G denotes a subset of PARTITIONS(Y), a, b, c, u denote elements of $Boolean^Y$, and P_1 denotes a partition of Y.

We now state a number of propositions:

(1) $a \Rightarrow b \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(2) $\forall_{a,P_1} G \wedge \forall_{b,P_1} G \Subset a \wedge b$.

(3) $a \wedge b \Subset \exists_{a,P_1} G \wedge \exists_{b,P_1} G$.

(4) $\neg(\forall_{a,P_1} G \wedge \forall_{b,P_1} G) = \exists_{\neg a,P_1} G \vee \exists_{\neg b,P_1} G$.

(5) $\neg(\exists_{a,P_1} G \wedge \exists_{b,P_1} G) = \forall_{\neg a,P_1} G \vee \forall_{\neg b,P_1} G$.

(6) $\forall_{a,P_1} G \vee \forall_{b,P_1} G \Subset a \vee b$.

(7) $a \vee b \Subset \exists_{a,P_1} G \vee \exists_{b,P_1} G$.

(8) $a \oplus b \Subset \neg(\exists_{\neg a,P_1} G \oplus \exists_{b,P_1} G) \vee \neg(\exists_{a,P_1} G \oplus \exists_{\neg b,P_1} G)$.

(9) $\forall_{a \vee b,P_1} G \Subset \forall_{a,P_1} G \vee \exists_{b,P_1} G$.

(10) $\forall_{a \vee b,P_1} G \Subset \exists_{a,P_1} G \vee \forall_{b,P_1} G$.

(11) $\forall_{a \vee b,P_1} G \Subset \exists_{a,P_1} G \vee \exists_{b,P_1} G$.

(12) $\exists_{a,P_1} G \wedge \forall_{b,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(13) $\forall_{a,P_1} G \wedge \exists_{b,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(14) $\forall_{a \Rightarrow b,P_1} G \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(15) $\forall_{a \Rightarrow b,P_1} G \Subset \exists_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(16) $\exists_{a,P_1} G \Rightarrow \forall_{b,P_1} G \Subset \forall_{a \Rightarrow b,P_1} G$.

(17) $a \Rightarrow b \Subset a \Rightarrow \exists_{b,P_1} G$.

(18) $a \Rightarrow b \Subset \forall_{a,P_1} G \Rightarrow b$.

(19) $\exists_{a \Rightarrow b,P_1} G \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(20) $\forall_{a,P_1} G \Subset \exists_{b,P_1} G \Rightarrow \exists_{a \wedge b,P_1} G$.

(21) If u is independent of P_1, G, then $\exists_{u \Rightarrow a,P_1} G \Subset u \Rightarrow \exists_{a,P_1} G$.

(22) If u is independent of P_1, G, then $\exists_{a \Rightarrow u,P_1} G \Subset \forall_{a,P_1} G \Rightarrow u$.

(23) $\forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G = \exists_{a \Rightarrow b,P_1} G$.

(24) $\forall_{a,P_1} G \Rightarrow \forall_{b,P_1} G \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(25) $\exists_{a,P_1} G \Rightarrow \exists_{b,P_1} G \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(26) $\forall_{a \Rightarrow b,P_1} G = \forall_{\neg a \vee b,P_1} G$.

(27) $\forall_{a \Rightarrow b,P_1} G = \neg \exists_{a \wedge \neg b,P_1} G$.

(28) $\exists_{a,P_1} G \Subset \neg(\forall_{a \Rightarrow b,P_1} G \wedge \forall_{a \Rightarrow \neg b,P_1} G)$.

(29) $\exists_{a,P_1} G \Subset \neg(\neg \exists_{a \wedge b,P_1} G \wedge \neg \exists_{a \wedge \neg b,P_1} G)$.

(30) $\exists_{a,P_1} G \wedge \forall_{a \Rightarrow b,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(31) $\exists_{a,P_1} G \wedge \neg \exists_{a \wedge b,P_1} G \Subset \neg \forall_{a \Rightarrow b,P_1} G$.

(32) $\forall_{a \Rightarrow c,P_1} G \wedge \forall_{c \Rightarrow b,P_1} G \Subset \forall_{a \Rightarrow b,P_1} G$.

(33) $\forall_{c \Rightarrow b,P_1} G \wedge \exists_{a \wedge c,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(34) $\forall_{b \Rightarrow \neg c,P_1} G \wedge \forall_{a \Rightarrow c,P_1} G \Subset \forall_{a \Rightarrow \neg b,P_1} G$.

(35) $\forall_{b \Rightarrow c,P_1} G \wedge \forall_{a \Rightarrow \neg c,P_1} G \Subset \forall_{a \Rightarrow \neg b,P_1} G$.

(36) $\forall_{b \Rightarrow \neg c,P_1} G \wedge \exists_{a \wedge c,P_1} G \Subset \exists_{a \wedge \neg b,P_1} G$.

(37) $\forall_{b \Rightarrow c,P_1} G \wedge \exists_{a \wedge \neg c,P_1} G \Subset \exists_{a \wedge \neg b,P_1} G$.

(38) $\exists_{c,P_1} G \wedge \forall_{c \Rightarrow b,P_1} G \wedge \forall_{c \Rightarrow a,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(39) $\forall_{b \Rightarrow c,P_1} G \wedge \forall_{c \Rightarrow \neg a,P_1} G \Subset \forall_{a \Rightarrow \neg b,P_1} G$.

(40) $\exists_{b,P_1} G \wedge \forall_{b \Rightarrow c,P_1} G \wedge \forall_{c \Rightarrow a,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

(41) $\exists_{c,P_1} G \wedge \forall_{b \Rightarrow \neg c,P_1} G \wedge \forall_{c \Rightarrow a,P_1} G \Subset \exists_{a \wedge \neg b,P_1} G$.

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