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Predicate Calculus for Boolean Valued Functions. Part XII

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The articles [11], [10], [2], [13], [8], [14], [1], [12], [3], [4], [15], [9], [6], [5], and [7] provide the notation and terminology for this paper.

For simplicity, we adopt the following rules: Y is a non empty set, G is a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M are partitions of Y, and x, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 are sets.

We now state a number of propositions:

(1) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(A,G) = B \wedge C \wedge D \wedge E \wedge F \wedge J$.

(2) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(B,G) = A \wedge C \wedge D \wedge E \wedge F \wedge J$.

(3) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(C,G) = A \wedge B \wedge D \wedge E \wedge F \wedge J$.

(4) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(D,G) = A \wedge B \wedge C \wedge E \wedge F \wedge J$.

(5) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(E,G) = A \wedge B \wedge C \wedge D \wedge F \wedge J$.

(6) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(F,G) = A \wedge B \wedge C \wedge D \wedge E \wedge J$.

(7) Suppose that $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(J,G) = A \wedge B \wedge C \wedge D \wedge E \wedge F$.

(8) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. Suppose that $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (F \dot{\longmapsto} F') {+\cdot} (J \dot{\longmapsto} J') {+\cdot} (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$.

(9) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. If $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (F \dot{\longmapsto} F') {+\cdot} (J \dot{\longmapsto} J') {+\cdot} (A \dot{\longmapsto} A')$, then $\mathrm{dom}\, h = \{A,B,C,D,E,F,J\}$.

(10) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. If $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (F \dot{\longmapsto} F') {+\cdot} (J \dot{\longmapsto} J') {+\cdot} (A \dot{\longmapsto} A')$, then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J)\}$.

(11) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is independent and $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J)$ meets $\mathrm{EqClass}(z, A)$.

(12) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J be partitions of Y, and z, u be elements of Y. Suppose that G is independent and $G = \{A,B,C,D,E,F,J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A,G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B,G))$.

(13) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(A,G) = B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(14) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(B,G) = A \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(15) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(C,G) = A \wedge B \wedge D \wedge E \wedge F \wedge J \wedge M$.

(16) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and

$B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(D,G) = A \wedge B \wedge C \wedge E \wedge F \wedge J \wedge M$.

(17) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(E,G) = A \wedge B \wedge C \wedge D \wedge F \wedge J \wedge M$.

(18) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(F,G) = A \wedge B \wedge C \wedge D \wedge E \wedge J \wedge M$.

(19) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(J,G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge M$.

(20) Suppose that $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(M,G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J$.

(21) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. Suppose that $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $h = (B \stackrel{.}{\longmapsto} B') +\cdot (C \stackrel{.}{\longmapsto} C') +\cdot (D \stackrel{.}{\longmapsto} D') +\cdot (E \stackrel{.}{\longmapsto} E') +\cdot (F \stackrel{.}{\longmapsto} F') +\cdot (J \stackrel{.}{\longmapsto} J') +\cdot (M \stackrel{.}{\longmapsto} M') +\cdot (A \stackrel{.}{\longmapsto} A')$. Then $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$.

(22) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. If $h = (B \stackrel{.}{\longmapsto} B') +\cdot (C \stackrel{.}{\longmapsto} C') +\cdot (D \stackrel{.}{\longmapsto} D') +\cdot (E \stackrel{.}{\longmapsto} E') +\cdot (F \stackrel{.}{\longmapsto} F') +\cdot (J \stackrel{.}{\longmapsto} J') +\cdot (M \stackrel{.}{\longmapsto} M') +\cdot (A \stackrel{.}{\longmapsto} A')$, then $\mathrm{dom}\, h = \{A,B,C,D,E,F,J,M\}$.

(23) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. Suppose $h = (B \stackrel{.}{\longmapsto} B') +\cdot (C \stackrel{.}{\longmapsto} C') +\cdot (D \stackrel{.}{\longmapsto} D') +\cdot (E \stackrel{.}{\longmapsto} E') +\cdot (F \stackrel{.}{\longmapsto} F') +\cdot (J \stackrel{.}{\longmapsto} J') +\cdot (M \stackrel{.}{\longmapsto} M') +\cdot (A \stackrel{.}{\longmapsto} A')$. Then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J), h(M)\}$.

(24) Let a be an element of $\mathit{Boolean}^Y$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is independent and $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(25) Let a be an element of $\mathit{Boolean}^Y$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M be partitions of Y, and z, u be elements of Y. Suppose that G is independent and $G = \{A,B,C,D,E,F,J,M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J \wedge M) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J \wedge M)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A,G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B,G))$.

The scheme *UI10* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, $\mathcal{I}$, $\mathcal{J}$ and a 10-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G},\mathcal{H},\mathcal{I},\mathcal{J}]$

provided the parameters meet the following condition:

- For all sets $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$ holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}]$.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9. The functor $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$ yielding a set is defined by:

(Def. 1) $x \in \{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$ or $x = x_9$.

One can prove the following propositions:

(26) If $x \in \{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$ or $x = x_9$.

(27) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1\} \cup \{x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$.

(28) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2\} \cup \{x_3,x_4,x_5,x_6,x_7,x_8,x_9\}$.

(29) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3\} \cup \{x_4,x_5,x_6,x_7,x_8,x_9\}$.

(30) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3,x_4\} \cup \{x_5,x_6,x_7,x_8,x_9\}$.

(31) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3,x_4,x_5\} \cup \{x_6,x_7,x_8,x_9\}$.

(32) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3,x_4,x_5,x_6\} \cup \{x_7,x_8,x_9\}$.

(33) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} \cup \{x_8,x_9\}$.

(34) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\} = \{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} \cup \{x_9\}$.

(35) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then CompF$(A,G) = B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(36) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then CompF$(B,G) = A \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(37) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then CompF$(C,G) = A \wedge B \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(38) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then CompF$(D,G) = A \wedge B \wedge C \wedge E \wedge F \wedge J \wedge M \wedge N$.

(39) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D \wedge F \wedge J \wedge M \wedge N$.

(40) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(F, G) = A \wedge B \wedge C \wedge D \wedge E \wedge J \wedge M \wedge N$.

(41) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(J, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge M \wedge N$.

(42) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(M, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge N$.

(43) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(N, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(44) Let A, B, C, D, E, F, J, M, N be sets, h be a function, and A', B', C', D', E', F', J', M', N' be sets. Suppose that $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (J \dot{\longmapsto} J') +\cdot (M \dot{\longmapsto} M') +\cdot (N \dot{\longmapsto} N') +\cdot (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$ and $h(M) = M'$ and $h(N) = N'$.

(45) Let $A, B, C, D, E, F, J, M, N$ be sets, h be a function, and $A', B', C', D', E', F', J', M', N'$ be sets. If $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (J \dot{\longmapsto} J') +\cdot (M \dot{\longmapsto} M') +\cdot (N \dot{\longmapsto} N') +\cdot (A \dot{\longmapsto}$ then $\mathrm{dom}\, h = \{A, B, C, D, E, F, J, M, N\}$.

(46) Let $A, B, C, D, E, F, J, M, N$ be sets, h be a function, and $A', B', C', D', E', F', J', M', N'$ be sets. Suppose $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (J \dot{\longmapsto} J') +\cdot (M \dot{\longmapsto} M') +\cdot (N \dot{\longmapsto} N') +$ Then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J), h(M), h(N)\}$.

(47) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J, M, N be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is independent and $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(48) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J, M, N be partitions of Y, and z, u be elements of Y. Suppose that G is independent and $G = \{A,B,C,D,E,F,J,M,N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B, G))$.

(49) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$ or $x = x_9$, then $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

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