

Five Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The articles [11], [10], [2], [12], [8], [13], [1], [3], [4], [9], [6], [5], and [7] provide the notation and terminology for this paper.

We follow the rules: Y is a non empty set, G is a subset of $\text{PARTITIONS}(Y)$, and A, B, C, D, E are partitions of Y .

We now state a number of propositions:

- (1) If $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$, then $\text{CompF}(A, G) = B \wedge C \wedge D \wedge E$.
- (2) Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\text{CompF}(B, G) = A \wedge C \wedge D \wedge E$.
- (3) Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\text{CompF}(C, G) = A \wedge B \wedge D \wedge E$.
- (4) Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\text{CompF}(D, G) = A \wedge B \wedge C \wedge E$.
- (5) Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\text{CompF}(E, G) = A \wedge B \wedge C \wedge D$.
- (6) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. Suppose $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $h = (B \mapsto B') + (C \mapsto C') + (D \mapsto D') + (E \mapsto E') + (A \mapsto A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$.
- (7) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. If $h = (B \mapsto B') + (C \mapsto C') + (D \mapsto D') + (E \mapsto E') + (A \mapsto A')$, then $\text{dom } h = \{A, B, C, D, E\}$.

- (8) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. If $h = (B \mapsto B') + (C \mapsto C') + (D \mapsto D') + (E \mapsto E') + (A \mapsto A')$, then $\text{rng } h = \{h(A), h(B), h(C), h(D), h(E)\}$.
- (9) Let G be a subset of $\text{PARTITIONS}(Y)$, A, B, C, D, E be partitions of Y , z, u be elements of Y , and h be a function. Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\text{EqClass}(u, B \wedge C \wedge D \wedge E)$ meets $\text{EqClass}(z, A)$.
- (10) Let G be a subset of $\text{PARTITIONS}(Y)$, A, B, C, D, E be partitions of Y , and z, u be elements of Y . Suppose that G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $\text{EqClass}(z, C \wedge D \wedge E) = \text{EqClass}(u, C \wedge D \wedge E)$. Then $\text{EqClass}(u, \text{CompF}(A, G))$ meets $\text{EqClass}(z, \text{CompF}(B, G))$.

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