JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Four Variable Predicate Calculus for Boolean Valued Functions. Part I

Shunichi Kobayashi
Ueda Multimedia Information Center
Nagano

Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

MML Identifier: BVFUNC20.

WWW: http://mizar.org/JFM/Vol11/bvfunc20.html

The articles [11], [10], [2], [13], [8], [14], [1], [12], [3], [4], [16], [15], [9], [6], [5], and [7] provide the notation and terminology for this paper.

1. PRELIMINARIES

For simplicity, we adopt the following convention: Y denotes a non empty set, a denotes an element of $Boolean^Y$, G denotes a subset of PARTITIONS(Y), and A, B, C, D denote partitions of Y.

One can prove the following propositions:

(1) Let h be a function and A', B', C', D' be sets. Suppose $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $h = (B \dot{\longmapsto} B') + \cdot (C \dot{\longmapsto} C') + \cdot (D \dot{\longmapsto} D') + \cdot (A \dot{\longmapsto} A')$. Then $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$.

(2) Let A, B, C, D be sets, h be a function, and A', B', C', D' be sets. If $h = (B \dot{\longmapsto} B') + \cdot (C \dot{\longmapsto} C') + \cdot (D \dot{\longmapsto} D') + \cdot (A \dot{\longmapsto} A')$, then $\mathrm{dom}\, h = \{A,B,C,D\}$.

(3) For every function h and for all sets A', B', C', D' such that $G = \{A,B,C,D\}$ and $h = (B \dot{\longmapsto} B') + \cdot (C \dot{\longmapsto} C') + \cdot (D \dot{\longmapsto} D') + \cdot (A \dot{\longmapsto} A')$ holds $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D)\}$.

(4) Let z, u be elements of Y and h be a function. Suppose G is independent and $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D)$ meets $\mathrm{EqClass}(z, A)$.

(5) Let z, u be elements of Y. Suppose G is independent and $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $\mathrm{EqClass}(z, C \wedge D) = \mathrm{EqClass}(u, C \wedge D)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B, G))$.

2. FOUR VARIABLE PREDICATE CALCULUS

We now state two propositions:

(20)[1] If G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \forall_{a,A} G, B} G \Subset \neg \forall_{\forall_{a,B} G, A} G$.

(23)[2] If G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{\neg a,A} G, B} G \Subset \neg \forall_{\forall_{a,B} G, A} G$.

REFERENCES

[1] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[2] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[3] Czesław Byliński. A classical first order language. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/cqc_lang.html.

[4] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/funct_4.html.

[5] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_1.html.

[6] Shunichi Kobayashi and Kui Jia. A theory of partitions. Part I. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/partit1.html.

[7] Shunichi Kobayashi and Yatsuka Nakamura. A theory of Boolean valued functions and quantifiers with respect to partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_2.html.

[8] Beata Padlewska. Families of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/setfam_1.html.

[9] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/eqrel_1.html.

[10] Andrzej Trybulec. Enumerated sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/enumset1.html.

[11] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[12] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fraenkel.html.

[13] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[14] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[15] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/valuat_1.html.

[16] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/margrel1.html.

Received November 26, 1999

Published February 3, 2003

---

[1] The propositions (6)–(19) have been removed.

[2] The propositions (21) and (22) have been removed.