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Predicate Calculus for Boolean Valued Functions. Part VI

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [11], [10], [2], [12], [13], [1], [8], [9], [3], [6], [5], [7], and [4] provide the notation and terminology for this paper.

We adopt the following rules: Y is a non empty set, G is a subset of PARTITIONS(Y), and A, B, C, D are partitions of Y.

One can prove the following propositions:

(1) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1 \wedge P_2) = \mathrm{EqClass}(z, P_1) \cap \mathrm{EqClass}(z, P_2)$.

(2) If $G = \{A, B\}$ and $A \neq B$, then $\bigwedge G = A \wedge B$.

(3) If $G = \{B, C, D\}$ and $B \neq C$ and $C \neq D$ and $D \neq B$, then $\bigwedge G = B \wedge C \wedge D$.

(4) If $G = \{A, B, C\}$ and $A \neq B$ and $C \neq A$, then $\mathrm{CompF}(A, G) = B \wedge C$.

(5) If $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$, then $\mathrm{CompF}(B, G) = C \wedge A$.

(6) If $G = \{A, B, C\}$ and $B \neq C$ and $C \neq A$, then $\mathrm{CompF}(C, G) = A \wedge B$.

(7) If $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$, then $\mathrm{CompF}(A, G) = B \wedge C \wedge D$.

(8) If $G = \{A, B, C, D\}$ and $A \neq B$ and $B \neq C$ and $B \neq D$, then $\mathrm{CompF}(B, G) = A \wedge C \wedge D$.

(9) If $G = \{A, B, C, D\}$ and $A \neq C$ and $B \neq C$ and $C \neq D$, then $\mathrm{CompF}(C, G) = A \wedge B \wedge D$.

(10) If $G = \{A, B, C, D\}$ and $A \neq D$ and $B \neq D$ and $C \neq D$, then $\mathrm{CompF}(D, G) = A \wedge C \wedge B$.

(14)[1] For all sets B, C, D, b, c, d holds $\mathrm{dom}((B \dot{\longmapsto} b) {+\cdot} (C \dot{\longmapsto} c) {+\cdot} (D \dot{\longmapsto} d)) = \{B, C, D\}$.

(15) For every function f and for all sets C, D, c, d such that $C \neq D$ holds $(f {+\cdot} (C \dot{\longmapsto} c) {+\cdot} (D \dot{\longmapsto} d))(C) = c$.

(16) For all sets B, C, D, b, c, d such that $B \neq C$ and $D \neq B$ holds $((B \dot{\longmapsto} b) {+\cdot} (C \dot{\longmapsto} c) {+\cdot} (D \dot{\longmapsto} d))(B) = b$.

[1] The propositions (11)–(13) have been removed.

(17) For all sets B, C, D, b, c, d and for every function h such that $h = (B \dot{\longmapsto} b) +\cdot (C \dot{\longmapsto} c) +\cdot (D \dot{\longmapsto} d)$ holds $\operatorname{rng} h = \{h(B), h(C), h(D)\}$.

(18) Let h be a function and A', B', C', D' be sets. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (A \dot{\longmapsto} A')$. Then $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$.

(19) Let A, B, C, D be sets, h be a function, and A', B', C', D' be sets. If $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (A \dot{\longmapsto} A')$, then $\operatorname{dom} h = \{A, B, C, D\}$.

(20) For every function h and for all sets A', B', C', D' such that $G = \{A, B, C, D\}$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (A \dot{\longmapsto} A')$ holds $\operatorname{rng} h = \{h(A), h(B), h(C), h(D)\}$.

(21) Let z, u be elements of Y and h be a function. Suppose G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D)$ meets $\mathrm{EqClass}(z, A)$.

(22) Let z, u be elements of Y. Suppose G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $\mathrm{EqClass}(z, C \wedge D) = \mathrm{EqClass}(u, C \wedge D)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B, G))$.

(23) Let z, u be elements of Y. Suppose G is independent and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $\mathrm{EqClass}(z, C) = \mathrm{EqClass}(u, C)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B, G))$.

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