Predicate Calculus for Boolean Valued Functions. Part IV

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

MML Identifier: BVFUNC12.

WWW: http://mizar.org/JFM/Vol11/bvfunc12.html

The articles [5], [7], [6], [4], [3], [1], and [2] provide the notation and terminology for this paper. In this paper *Y* denotes a non empty set. The following propositions are true:

- (5)¹ For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\neg \forall_{\exists_{a,A}G,B}G = \exists_{\forall_{\neg a,A}G,B}G$.
- (6) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\neg \exists_{\forall_{a,A}G,B}G = \forall_{\exists_{\neg a,A}G,B}G$.
- (7) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\neg \forall_{\forall_{a,A}G,B}G = \exists_{\exists_{\neg_{a,A}G,B}G}$.
- (11)² Let *a* be an element of *Boolean*^{*Y*}, *G* be a subset of PARTITIONS(*Y*), and *A*, *B* be partitions of *Y*. If *G* is independent, then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.
- (12) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,A}G,B}G$.
- (13) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,A}G,B}G$.
- (14) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.
- (15) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.
- (16) For every element *a* of *Boolean*^{*Y*} and for every subset *G* of PARTITIONS(*Y*) and for all partitions *A*, *B* of *Y* holds $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.

¹ The propositions (1)–(4) have been removed.

² The propositions (8)–(10) have been removed.

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Received August 17, 1999

Published January 2, 2004