JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Propositional Calculus for Boolean Valued Functions. Part VI

Shunichi Kobayashi
Shinshu University
Nagano

Summary. In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

MML Identifier: BVFUNC10.

WWW: http://mizar.org/JFM/Vol11/bvfunc10.html

The articles [3], [5], [4], [2], and [1] provide the notation and terminology for this paper.

In this paper Y denotes a non empty set.

One can prove the following propositions:

(1) For all elements a, b, c of $Boolean^Y$ holds $a \wedge b \vee b \wedge c \vee c \wedge a = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

(2) For all elements a, b, c of $Boolean^Y$ holds $a \wedge \neg b \vee b \wedge \neg c \vee c \wedge \neg a = b \wedge \neg a \vee c \wedge \neg b \vee a \wedge \neg c$.

(3) For all elements a, b, c of $Boolean^Y$ holds $(a \vee \neg b) \wedge (b \vee \neg c) \wedge (c \vee \neg a) = (b \vee \neg a) \wedge (c \vee \neg b) \wedge (a \vee \neg c)$.

(4) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow a = true(Y)$ and $c \Rightarrow b = true(Y)$ holds $c \Rightarrow a \vee b = true(Y)$.

(5) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow c = true(Y)$ and $b \Rightarrow c = true(Y)$ holds $a \wedge b \Rightarrow c = true(Y)$.

(6) For all elements a_1, a_2, b_1, b_2, c_1, c_2 of $Boolean^Y$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \Subset a_2 \vee b_2 \vee c_2$.

(7) For all elements a_1, a_2, b_1, b_2 of $Boolean^Y$ holds $(a_1 \Rightarrow b_1) \wedge (a_2 \Rightarrow b_2) \wedge (a_1 \vee a_2) \wedge \neg(b_1 \wedge b_2) = (b_1 \Rightarrow a_1) \wedge (b_2 \Rightarrow a_2) \wedge (b_1 \vee b_2) \wedge \neg(a_1 \wedge a_2)$.

(8) For all elements a, b, c, d of $Boolean^Y$ holds $(a \vee b) \wedge (c \vee d) = a \wedge c \vee a \wedge d \vee b \wedge c \vee b \wedge d$.

(9) For all elements a_1, a_2, b_1, b_2, b_3 of $Boolean^Y$ holds $a_1 \wedge a_2 \vee b_1 \wedge b_2 \wedge b_3 = (a_1 \vee b_1) \wedge (a_1 \vee b_2) \wedge (a_1 \vee b_3) \wedge (a_2 \vee b_1) \wedge (a_2 \vee b_2) \wedge (a_2 \vee b_3)$.

(10) For all elements a, b, c, d of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow d) = (a \Rightarrow b \wedge c \wedge d) \wedge (b \Rightarrow c \wedge d) \wedge (c \Rightarrow d)$.

(11) For all elements a, b, c, d of $Boolean^Y$ holds $(a \Rightarrow c) \wedge (b \Rightarrow d) \wedge (a \vee b) \Subset c \vee d$.

(12) For all elements a, b, c of $Boolean^Y$ holds $(a \wedge b \Rightarrow \neg c) \wedge a \wedge c \Subset \neg b$.

(13) For all elements a_1, a_2, a_3, b_1, b_2, b_3 of $Boolean^Y$ holds $a_1 \wedge a_2 \wedge a_3 \Rightarrow b_1 \vee b_2 \vee b_3 = \neg b_1 \wedge \neg b_2 \wedge a_3 \Rightarrow \neg a_1 \vee \neg a_2 \vee b_3$.

(14) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) = a \wedge b \wedge c \vee \neg a \wedge \neg b \wedge \neg c$.

(15) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) \wedge (a \vee b \vee c) = a \wedge b \wedge c$.

(16) For all elements a, b, c of $Boolean^Y$ holds $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) \wedge \neg(a \wedge b \wedge c) = \neg a \wedge b \wedge c \vee a \wedge \neg b \wedge c \vee a \wedge b \wedge \neg c$.

(17) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \wedge c$.

(18) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \vee b \Rightarrow c$.

(19) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee c$.

(20) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee \neg c$.

(21) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee a$.

(22) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee \neg a$.

(23) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b) \wedge (b \Rightarrow c \vee a)$.

(24) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c)$.

(25) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee c) \wedge (b \Rightarrow c \vee a)$.

(26) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee a)$.

(27) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee \neg a)$.

REFERENCES

[1] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Journal of Formalized Mathematics*, 10, 1998. `http://mizar.org/JFM/Vol10/bvfunc_1.html`.

[2] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/fraenkel.html`.

[3] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[4] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/valuat_1.html`.

[5] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/margrel1.html`.

Received July 14, 1999

Published January 2, 2004