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Preliminaries to Arithmetic

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The articles [4], [2], [7], [1], [5], [6], and [3] provide the notation and terminology for this paper.

1. Main Part

Let r be a number. We say that r is real if and only if:

(Def. 1) $r \in \mathbb{R}$.

Let us observe that there exists a number which is real.

Let x, y be real numbers. The functor $x+y$ is defined by:

(Def. 2)(i) There exist elements x', y' of $REAL+$ such that $x = x'$ and $y = y'$ and $x+y = x'+y'$ if $x \in REAL+$ and $y \in REAL+$,

(ii) there exist elements x', y' of $REAL+$ such that $x = x'$ and $y = \langle \emptyset, y' \rangle$ and $x+y = x'-y'$ if $x \in REAL+$ and $y \in [: \{\emptyset\}, REAL+ :]$,

(iii) there exist elements x', y' of $REAL+$ such that $x = \langle \emptyset, x' \rangle$ and $y = y'$ and $x+y = y'-x'$ if $y \in REAL+$ and $x \in [: \{\emptyset\}, REAL+ :]$,

(iv) there exist elements x', y' of $REAL+$ such that $x = \langle \emptyset, x' \rangle$ and $y = \langle \emptyset, y' \rangle$ and $x+y = \langle \emptyset, y'+x' \rangle$, otherwise.

Let us observe that the functor $x+y$ is commutative. The functor $x \cdot y$ is defined by:

(Def. 3)(i) There exist elements x', y' of $REAL+$ such that $x = x'$ and $y = y'$ and $x \cdot y = x' * y'$ if $x \in REAL+$ and $y \in REAL+$,

(ii) there exist elements x', y' of $REAL+$ such that $x = x'$ and $y = \langle \emptyset, y' \rangle$ and $x \cdot y = \langle \emptyset, x' * y' \rangle$ if $x \in REAL+$ and $y \in [: \{\emptyset\}, REAL+ :]$ and $x \neq \emptyset$,

(iii) there exist elements x', y' of $REAL+$ such that $x = \langle \emptyset, x' \rangle$ and $y = y'$ and $x \cdot y = \langle \emptyset, y' * x' \rangle$ if $y \in REAL+$ and $x \in [: \{\emptyset\}, REAL+ :]$ and $y \neq \emptyset$,

(iv) there exist elements x', y' of $REAL+$ such that $x = \langle \emptyset, x' \rangle$ and $y = \langle \emptyset, y' \rangle$ and $x \cdot y = y' * x'$ if $x \in [: \{\emptyset\}, REAL+ :]$ and $y \in [: \{\emptyset\}, REAL+ :]$,

(v) $x \cdot y = \emptyset$, otherwise.

Let us observe that the functor $x \cdot y$ is commutative. The predicate $x \leq y$ is defined by:

(Def. 4)(i) There exist elements x', y' of $REAL+$ such that $x = x'$ and $y = y'$ and $x' \leq y'$ if $x \in REAL+$ and $y \in REAL+$,

(ii) there exist elements x', y' of $REAL+$ such that $x = \langle \emptyset, x' \rangle$ and $y = \langle \emptyset, y' \rangle$ and $y' \leq x'$ if $x \in [: \{\emptyset\}, REAL+ :]$ and $y \in [: \{\emptyset\}, REAL+ :]$,

(iii) $y \in REAL+$ and $x \in [: \{\emptyset\}, REAL+ :]$, otherwise.

Let us notice that the predicate $x \leq y$ is reflexive and connected. We introduce $y \geq x$ as a synonym of $x \leq y$. We introduce $y < x$ and $x > y$ as antonyms of $x \leq y$.

Let x, y be real numbers. Note that $x + y$ is real and $x \cdot y$ is real.

Let us note that every element of $\mathbb{R}$ is real.

Let x, y be elements of $\mathbb{R}$. Then $x + y$ is an element of $\mathbb{R}$. Then $x \cdot y$ is an element of $\mathbb{R}$.

Let us note that every number which is natural is also real.

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