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Fix Point Theorem for Compact Spaces

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Summary. The Banach theorem in compact metric spaces is proved.

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The articles [10], [11], [1], [5], [8], [7], [12], [3], [9], [4], [2], and [6] provide the notation and terminology for this paper.

In this paper M denotes a non empty metric space.

One can prove the following proposition

(1) Let F be a set. Suppose F is finite and $F \neq \emptyset$ and F is $\subseteq$-linear. Then there exists a set m such that $m \in F$ and for every set C such that $C \in F$ holds $m \subseteq C$.

Let M be a non empty metric space. A function from the carrier of M into the carrier of M is said to be a contraction of M if:

(Def. 1) There exists a real number L such that $0 < L$ and $L < 1$ and for all points x, y of M holds $\rho(\mathrm{it}(x), \mathrm{it}(y)) \leq L \cdot \rho(x, y)$.

Next we state the proposition

(2) Let f be a contraction of M. Suppose M_{top} is compact. Then there exists a point c of M such that $f(c) = c$ and for every point x of M such that $f(x) = x$ holds $x = c$.

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