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Construction of Finite Sequence over Ring and Left-, Right-, and Bi-Modules over a Ring[1]

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Summary. This text includes definitions of finite sequences over rings and left-, right-, and bi-module over a ring treated as functions defined for *all* natural numbers, but with almost everywhere equal to zero. Some elementary theorems are proved, in particular for each category of sequences the scheme about existence is proved. In all four cases, i.e. for rings, left-, right and bi- modules are almost exactly the same, hovewer we do not know how to do the job in Mizar in a different way. The paper is mostly based on the paper [2]. In particular the notion of initial segment of natural numbers is introduced which differs from that of [2] by starting with zero. This proved to be more convenient for algebraic purposes.

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The articles [6], [9], [7], [1], [10], [3], [8], [4], and [5] provide the notation and terminology for this paper.

In this paper i, k, m, n are natural numbers.

Next we state two propositions:

(2)[1] If $m < n+1$, then $m < n$ or $m = n$.

(4)[2] If $k < 2$, then $k = 0$ or $k = 1$.

Let us consider n. The functor $\mathrm{PSeg}\, n$ yielding a set is defined as follows:

(Def. 1) $\mathrm{PSeg}\, n = \{k : k < n\}$.

Let us consider n. Then $\mathrm{PSeg}\, n$ is a subset of $\mathbb{N}$.

Next we state several propositions:

(10)[3] $k \in \mathrm{PSeg}\, n$ iff $k < n$.

(11) $\mathrm{PSeg}\, 0 = \emptyset$ and $\mathrm{PSeg}\, 1 = \{0\}$ and $\mathrm{PSeg}\, 2 = \{0, 1\}$.

(12) $n \in \mathrm{PSeg}(n+1)$.

(13) $n \leq m$ iff $\mathrm{PSeg}\, n \subseteq \mathrm{PSeg}\, m$.

[1] Supported by RPBP.III-24.C3.

[1] The proposition (1) has been removed.
[2] The proposition (3) has been removed.
[3] The propositions (5)–(9) have been removed.

(14) If $\mathrm{PSeg}\,n = \mathrm{PSeg}\,m$, then $n = m$.

(15) If $k \leq n$, then $\mathrm{PSeg}\,k = \mathrm{PSeg}\,k \cap \mathrm{PSeg}\,n$ and $\mathrm{PSeg}\,k = \mathrm{PSeg}\,n \cap \mathrm{PSeg}\,k$.

(16) If $\mathrm{PSeg}\,k = \mathrm{PSeg}\,k \cap \mathrm{PSeg}\,n$ or $\mathrm{PSeg}\,k = \mathrm{PSeg}\,n \cap \mathrm{PSeg}\,k$, then $k \leq n$.

(17) $\mathrm{PSeg}\,n \cup \{n\} = \mathrm{PSeg}(n+1)$.

In the sequel R is a non empty zero structure.

Let us consider R and let F be a sequence of R. We say that F is finite-Support if and only if:

(Def. 2) There exists n such that for every i such that $i \geq n$ holds $F(i) = 0_R$.

Let us consider R. Note that there exists a sequence of R which is finite-Support.

Let us consider R. An algebraic sequence of R is a finite-Support sequence of R.

In the sequel p, q denote algebraic sequences of R.

Let us consider R, p, k. We say that the length of p is at most k if and only if:

(Def. 3) For every i such that $i \geq k$ holds $p(i) = 0_R$.

Let us consider R, p. The functor $\mathrm{len}\,p$ yielding a natural number is defined as follows:

(Def. 4) The length of p is at most $\mathrm{len}\,p$ and for every m such that the length of p is at most m holds $\mathrm{len}\,p \leq m$.

Next we state three propositions:

(22)[4] If $i \geq \mathrm{len}\,p$, then $p(i) = 0_R$.

(24)[5] If for every i such that $i < k$ holds $p(i) \neq 0_R$, then $\mathrm{len}\,p \geq k$.

(25) If $\mathrm{len}\,p = k+1$, then $p(k) \neq 0_R$.

Let us consider R, p. The functor $\mathrm{support}\,p$ yielding a subset of $\mathbb{N}$ is defined by:

(Def. 5) $\mathrm{support}\,p = \mathrm{PSeg}\,\mathrm{len}\,p$.

The following proposition is true

(27)[6] $k = \mathrm{len}\,p$ iff $\mathrm{PSeg}\,k = \mathrm{support}\,p$.

The scheme *AlgSeqLambdaF* deals with a non empty zero structure $\mathcal{A}$, a natural number $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

There exists an algebraic sequence p of $\mathcal{A}$ such that $\mathrm{len}\,p \leq \mathcal{B}$ and for every k such that $k < \mathcal{B}$ holds $p(k) = \mathcal{F}(k)$

for all values of the parameters.

The following propositions are true:

(28) If $\mathrm{len}\,p = \mathrm{len}\,q$ and for every k such that $k < \mathrm{len}\,p$ holds $p(k) = q(k)$, then $p = q$.

(29) If the carrier of $R \neq \{0_R\}$, then for every k there exists an algebraic sequence p of R such that $\mathrm{len}\,p = k$.

In the sequel x is an element of R.

Let us consider R, x. The functor $\langle_0 x\rangle$ yielding an algebraic sequence of R is defined by:

(Def. 6) $\mathrm{len}\langle_0 x\rangle \leq 1$ and $\langle_0 x\rangle(0) = x$.

One can prove the following four propositions:

(31)[7] $p = \langle_0 0_R\rangle$ iff $\mathrm{len}\,p = 0$.

(32) $p = \langle_0 0_R\rangle$ iff $\mathrm{support}\,p = \emptyset$.

(33) $\langle_0 0_R\rangle(i) = 0_R$.

(34) $p = \langle_0 0_R\rangle$ iff $\mathrm{rng}\,p = \{0_R\}$.

[4] The propositions (18)–(21) have been removed.
[5] The proposition (23) has been removed.
[6] The proposition (26) has been removed.
[7] The proposition (30) has been removed.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[3] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[4] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_2.html`.

[5] Jan Popiołek. Real normed space. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/normsp_1.html`.

[6] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[7] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

[8] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/rlvect_1.html`.

[9] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[10] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

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