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One-Dimensional Congruence of Segments, Basic Facts and Midpoint Relation[1]

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Summary. We study a theory of one-dimensional congruence of segments. The theory is characterized by a suitable formal axiom system; as a model of this system one can take the structure obtained from any weak directed geometrical bundle, with the congruence interpreted as in the case of "classical" vectors. Preliminary consequences of our axiom system are proved, basic relations of maximal distance and of midpoint are defined, and several fundamental properties of them are established.

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The articles [4], [2], [5], [3], [6], and [1] provide the notation and terminology for this paper.

Let A be a non empty set and let C be a binary relation on $[:A, A:]$. Observe that $\langle A, C \rangle$ is non empty.

Let I_1 be a non empty affine structure. We say that I_1 is weak segment-congruence space-like if and only if the conditions (Def. 2) are satisfied.

(Def. 2)[1] For all elements a, b of I_1 holds $a,b \downharpoonleft\!\upharpoonright b,a$ and for all elements a, b of I_1 such that $a,b \downharpoonleft\!\upharpoonright a,a$ holds $a = b$ and for all elements a, b, c, d, p, q of I_1 such that $a,b \downharpoonleft\!\upharpoonright p,q$ and $c,d \downharpoonleft\!\upharpoonright p,q$ holds $a,b \downharpoonleft\!\upharpoonright c,d$ and for all elements a, c of I_1 there exists an element b of the carrier of I_1 such that $a,b \downharpoonleft\!\upharpoonright b,c$ and for all elements a, a', b, b', p of I_1 such that $a \neq a'$ and $b \neq b'$ and $p,a \downharpoonleft\!\upharpoonright p,a'$ and $p,b \downharpoonleft\!\upharpoonright p,b'$ holds $a,b \downharpoonleft\!\upharpoonright a',b'$ and for all elements a, b of I_1 holds $a = b$ or there exists an element c of I_1 such that $a \neq c$ and $a,b \downharpoonleft\!\upharpoonright b,c$ or there exist elements p, p' of I_1 such that $p \neq p'$ and $a,b \downharpoonleft\!\upharpoonright p,p'$ and $a,p \downharpoonleft\!\upharpoonright p,b$ and $a,p' \downharpoonleft\!\upharpoonright p',b$ and for all elements a, b, b', p, p', c of I_1 such that $a,b \downharpoonleft\!\upharpoonright b,c$ and $b,b' \downharpoonleft\!\upharpoonright p,p'$ and $b,p \downharpoonleft\!\upharpoonright p,b'$ and $b,p' \downharpoonleft\!\upharpoonright p',b'$ holds $a,b' \downharpoonleft\!\upharpoonright b',c$ and for all elements a, b, b', c of I_1 such that $a \neq c$ and $b \neq b'$ and $a,b \downharpoonleft\!\upharpoonright b,c$ and $a,b' \downharpoonleft\!\upharpoonright b',c$ there exist elements p, p' of I_1 such that $p \neq p'$ and $b,b' \downharpoonleft\!\upharpoonright p,p'$ and $b,p \downharpoonleft\!\upharpoonright p,b'$ and $b,p' \downharpoonleft\!\upharpoonright p',b'$ and for all elements a, b, c, p, p', q, q' of I_1 such that $a,b \downharpoonleft\!\upharpoonright p,p'$ and $a,c \downharpoonleft\!\upharpoonright q,q'$ and $a,p \downharpoonleft\!\upharpoonright p,b$ and $a,q \downharpoonleft\!\upharpoonright q,c$ and $a,p' \downharpoonleft\!\upharpoonright p',b$ and $a,q' \downharpoonleft\!\upharpoonright q',c$ there exist elements r, r' of I_1 such that $b,c \downharpoonleft\!\upharpoonright r,r'$ and $b,r \downharpoonleft\!\upharpoonright r,c$ and $b,r' \downharpoonleft\!\upharpoonright r',c$.

One can verify that there exists a non empty affine structure which is strict, non trivial, and weak segment-congruence space-like.

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[1] The definition (Def. 1) has been removed.

A weak segment-congruence space is a non trivial weak segment-congruence space-like non empty affine structure.

We adopt the following rules: A_1 is a weak segment-congruence space and a, b, b', b'', c, d, p, p' are elements of A_1.

The following propositions are true:

(1) $a,b \downharpoonleft\upharpoonright a,b$.

(2) If $a,b \downharpoonleft\upharpoonright c,d$, then $c,d \downharpoonleft\upharpoonright a,b$.

(3) If $a,b \downharpoonleft\upharpoonright c,d$, then $a,b \downharpoonleft\upharpoonright d,c$.

(4) If $a,b \downharpoonleft\upharpoonright c,d$, then $b,a \downharpoonleft\upharpoonright c,d$.

(5) For all a, b holds $a,a \downharpoonleft\upharpoonright b,b$.

(6) If $a,b \downharpoonleft\upharpoonright c,c$, then $a = b$.

(7) If $a,b \downharpoonleft\upharpoonright p,p'$ and $a,b \downharpoonleft\upharpoonright b,c$ and $a,p \downharpoonleft\upharpoonright p,b$ and $a,p' \downharpoonleft\upharpoonright p',b$, then $a = c$.

(8) If $a,b' \downharpoonleft\upharpoonright a,b''$ and $a,b \downharpoonleft\upharpoonright a,b''$, then $b = b'$ or $b = b''$ or $b' = b''$.

Let us consider A_1 and let us consider a, b. We say that a, b are in a maximal distance if and only if:

(Def. 4)[2] There exist p, p' such that $p \neq p'$ and $a,b \downharpoonleft\upharpoonright p,p'$ and $a,p \downharpoonleft\upharpoonright p,b$ and $a,p' \downharpoonleft\upharpoonright p',b$.

Let us consider A_1 and let us consider a, b, c. We say that b is a midpoint of a, c if and only if:

(Def. 5) $a = b$ and $b = c$ and $a = c$ or $a = c$ and a, b are in a maximal distance or $a \neq c$ and $a,b \downharpoonleft\upharpoonright b,c$.

Next we state three propositions:

(11)[3] If $a \neq b$ and a, b are not in a maximal distance, then there exists c such that $a \neq c$ and $a,b \downharpoonleft\upharpoonright b,c$.

(12) If a, b are in a maximal distance and $a,b \downharpoonleft\upharpoonright b,c$, then $a = c$.

(13) If a, b are in a maximal distance, then $a \neq b$.

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[2] The definition (Def. 3) has been removed.

[3] The propositions (9) and (10) have been removed.